Export and the Labor Market:  
a Dynamic Model with on-the-job Search

Davide Suverato *†

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Abstract

This paper develops a dynamic general equilibrium model of trade with heterogeneous firms and homogeneous workers who search for a job while being employed; on the job search (OJS). To this end, I adapt Mortensen’s (2010) model of the labor market with OJS, to a dynamic setup with firm entry, and embed this framework in a version of the Melitz (2003) trade model. The model features the fully tractable transitional dynamics of the labor market with unemployment and wage dispersion, while having the same aggregate properties as Melitz’s industry equilibrium.

A trade induced selection of least productive firms going out of the market causes a simultaneous destruction of jobs. In the short run, unemployment peaks and the probability to find a job falls. Over time, workers reallocate from shrinking low productive firms to expanding high productive firms, which pay higher wages and export. Indeed, during the transitional dynamics a trade-off exists between a sudden increase of aggregate productivity and a sluggish recovery from trade induced job losses. Only in the long run, the initial employment loss is offset, while the efficiency gains from trade remain.

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*LMU University of Munich, e-mail address: davide.suverato@econ.lmu.de
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1 Introduction

One issue at the core of the recent literature on international trade is the reallocation of workers after a trade liberalization. While comparative advantage motives have been proposed in the past to explain the reallocation of resources across sectors, evidence shows that inter–firm reallocation accounts for most of the labor market response to trade liberalization. Nevertheless, we know very little about the theoretical mechanisms behind trade–induced reallocation of workers across firms. Moreover, although there is consensus on long–run efficiency gains from trade, there is a lack of analysis on the short–run response and adjustment of the labor market to trade liberalization. The aim of this paper is to shed light on these aspects. The task involves two challenges. First, most trade models are static, or confined to a steady state analysis; while the effect of a trade liberalization on inter–firm worker reallocation is a dynamic phenomenon which is hardly taken into account if only steady state comparisons are considered. Second, labor market frictions and unemployment might be useful to characterize a sluggish adjustment of the labor market (as in recent trade models), but this is not sufficient to generate wage dispersion. And wage dispersion is instead a first order candidate to explain why workers reallocate across firms: simply, they search for better paid jobs.

In this paper I develop a dynamic general equilibrium model of trade with heterogeneous firms and homogeneous workers who search for jobs, both when they are unemployed and employed; so called on–the–job search (henceforth OJS). The labor market features a wage dispersion equilibrium with large firms and wage bargaining, along the lines of Mortensen (2010). Employed workers separate from their employer whenever they receive a better wage offer. Hence, firms have higher chances of hiring a worker (and not losing a current employee) if they offer a higher wage. This mechanism generates competition for workers among employers, and as a consequence a wage distribution arises in equilibrium. Firm heterogeneity with uncertainty on the profitability of entry is introduced in a similar way to Melitz (2003). In equilibrium more productive firms pay higher wages and make forward looking decisions on entry, exit, export and on the number of vacancies. A trade liberalization, by causing the exit of least productive firms, determines the destruction of jobs, with a simultaneous peak of unemployment and a reduction of the probability to find a job. Since exporters pay higher wages, over time more and more employees reallocate to expanding exporter firms, at the expenses of low productive firms that shrink (although they remain in the market). Non–exporters partially replace employees who separate by hiring more from the pool of unemployed, and more slowly relative to exporters. The exit of least productive incumbents leads to higher average profits which triggers more firm entry, and indeed job creation, during the transition. The composite effect of these three channels decreases unemployment and increases the probability to find a job. In the long run, the initial employment loss is offset, both average employment per firm and industry productivity are higher than before the trade liberalization and welfare gains from trade are unambiguous.

There is robust empirical evidence on both the positive effect of trade–induced firm exit on productivity growth and on the finding that exporters pay higher wages and employ more workers.\footnote{See Bernard et al. (2007) for an extensive review.} However, it is in relation to the predictions about job destruction, job creation and employment real-
location across firms that the model is able to reproduce the salient observed patterns of employment in response to trade liberalization. Trefler (2004) finds that the U.S.–Canada F.T.A. is associated with substantial job losses in the short run (up to 12% of the pre–reform industry employment) which the economy compensates for in the long run. Strongly connected to this paper is a whole strand of the literature examining effects from increased trade with China on the U.S.–labor market (e.g. Autor et al. (2014), Acemoglu et al. (2016) and Autor et al. (2016)). Levinsohn (1999), for Chile, finds that in the export sectors the largest 5% of plants account for more than 45% of all jobs created after a trade liberalization. Molina and Munendler (2013) document how firms actively hire workforce in preparation for exporting, and provide evidence that firms that become exporters poach workers from other firms. Davis et al. (2013) find that larger firms fill their vacancies faster, and Felbermayr et al. (2014) discuss how this finding extends to exporters, as a direct consequence of exporter size and the wage premium. Moreover, a large body of empirical research shows that labor market response to trade liberalization is characterized by substantial transitional dynamics and inter–firm reallocations. These findings motivate two lines of research which this paper aims to contribute to. First, few and recent trade models with heterogeneous firms account for dynamic adjustment after trade liberalization; these includes Ghironi and Melitz (2005) and Melitz and Burstein (2013). Their focus is on firm entry and innovation over the business cycle, however they abstract from labor market imperfections. Second, open economy models with labor market frictions, such as Cosar et al. (2013), Cacciatore (2014) and Dix-Carneiro (2014) among others, provide dynamic general equilibrium analysis of the response of employment to trade liberalization and quantify the effect over time. However, these papers do not allow for job-to-job mobility of workers across firms, which in my framework is a key feature of the labor market adjustment.

The modeling approach based on OJS is motivated by both theory and evidence on the labor market. Other recent papers extend the Melitz’s model with search and matching frictions; such as Helpman and Itskhoiki (2010) and Felbermayr et al. (2011a), who provide steady state analysis with wage bargaining, or Egger and Kreckemeier (2009) and Egger et al. (2013) whose approach is based on fair wages. These models all assume that only unemployed workers search. However, research in labor economics shows that the status of (or the threat of) unemployment plays a limited role, alone, in explaining worker reallocation. Moreover, empirical studies find robust evidence of this

\footnote{Felbermayr et al. (2011b) show that in the long run the effect of trade openness on employment vanishes after controlling for business cycle factors. If there is an effect of trade openness on long run unemployment it seems to be weak and confined to a reduction of unemployment for high–skill workers only.}

\footnote{Levinsohn (1999) documents that the reallocation of workers between sectors accounts for about 10% of total labor market response after trade liberalization in Chile. Wacziarg and Wallack (2004) estimate for a panel of 25 countries, that a trade liberalization episode increases inter-sectoral job reallocation by about 1.5%. In contrast, Haltiwanger et al. (2004) show that trade liberalization episodes in Latin American countries increase within-sector reallocation from 8.9% in Argentina to 16.4% in Brazil.

Recent studies by Schaal (2012) and Kaas and Kircher (2015) account for labor markets dynamics with heterogeneous firms, but they do not consider international trade integration.

In a very influential study, Calvà et al. (2006) estimate a general equilibrium model with strategic wage bargaining and on-the-job search for France. They find that the former source of employment explains more than half of the rise in wages above the worker’ reservation wage. The theoretical motivation for OJS can be found in Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002): OJS allows employed workers to put their employer into Bertrand price competition with alternative employers.
channel. Hall and Krueger (2012) show that about 40% of U.S. employed workers are searching while employed. Bjelland et al. (2011) show that 30% of separations each quarter are employer-to-employer reallocations. Fallick and Fleischman (2004) document that the number of monthly job-to-job transitions is twice as high as the flows of workers from employment to unemployment, and that nearly two-fifths of new jobs are due to employer changes. In a study of the Swedish labor market response to trade liberalization, Davidson et al. (2012) find that within ten years after the policy implementation 34% of workers were observed in at least two different firms, the median of job-to-job movers per firm is of 30 workers and only 3% of firms do not have job-to-job movers. This evidence is consistent with the idea that OJS is a main driver of worker reallocation during the adjustment of the labor market in response to trade liberalization.

Another recent paper by Fajgelbaum (2013) also introduces OJS into a trade model. He focuses on how labor market frictions affect the age at which firms become exporters. To this end, he assumes that revenue per worker does not depend on firm employment and he confines the labor market to a steady state analysis. In contrast, the present paper allows marginal revenue to be decreasing in employment and characterizes in full the dynamics of the labor market. The first feature is necessary to derive the equilibrium wage distribution. The second feature explains the reallocation of workers across firms as the tightness of the labor market evolves over time. Both channels are at the core of the model in this paper and are not addressed in Fajgelbaum (2013).

Two other recent working papers, Felbermayr et al. (2014) and Helpman and Itskhoki (2015), study the trade–induced reallocation of workers across heterogeneous firms in a dynamic setup with search frictions. Both these papers and mine predict that the response of labor market to trade liberalization takes time, exporters adjust employment faster than non–exporters, and low productivity firms that remain in the market cut wages on impact and increase them during the transition as they reduce employment. However, in their two studies the probability to find a job is constant over the labor market adjustment to trade liberalization. In contrast, in this paper OJS allows the job finding probability to respond to trade liberalization. The transitional dynamics of the job finding probability has an impact on both wage distribution and firm employment which evolve over time. This innovation allows the modeling framework to be well suited to the analysis of both efficiency gains and employment costs which arise in the dynamic adjustment of labor market after a trade liberalization.

The remainder of the paper is organized in four sections. The next section outlines the model: output market, industry dynamics and labor market dynamics. In Sections 3 and 4 I characterize the equilibrium of the labor market and the industry equilibrium. The last section concludes.

2 Setup of the model

There are two countries, home and foreign. In each country a continuum of single product monopolists produce varieties of a differentiated good which are mobile between countries. Time is discrete and indexed by $t = 0, 1, 2, \ldots$.

6These predictions are consistent with evidence based on recent empirical studies, such as Amiti and Davis (2012).
**Endowments.** The domestic and foreign countries are populated by a continuum of workers measured as \(N\) and \(N^*\) respectively. Each worker is endowed with one unit of homogeneous labor that he/she is willing to rent to firms inelastically in exchange for a given wage. Workers are immobile between countries.

**Preferences.** Consumption is allocated over a continuum of varieties indexed by \(i\) in the set of varieties available in the market \(\Omega\). In both countries preferences are represented by a C.E.S. utility function. Consumption allocation can be described by means of an aggregate good \(Q_t\) and a consumption based price index \(P_t\), such that \(P_t Q_t\) is total expenditure by domestic consumers on all varieties (domestic and foreign produced) sold in the domestic market:

\[
Q_t = \left[ \int_{\Omega} q_t(i) \frac{1}{\sigma} \, di \right]^{\frac{1}{1-\sigma}} , \quad P_t = \left[ \int_{\Omega} p_{qt} (i)^{1-\sigma} \, di \right]^{\frac{1}{1-\sigma}}
\]

where \(q_t(i)\) and \(p_t(i)\) are respectively quantity and price of the variety \(i\) of the consumption good, and \(\sigma > 1\) is the elasticity of substitution between any two varieties. There is no means to store wealth, therefore consumers do not transfer consumption across periods.

A domestic firm producing variety \(i\) sells the quantity \(q_t(i)\) in the domestic market at a price \(p_{qt}(i)\) and exports the quantity \(q^*_t(i)\) in the foreign market at a price \(p^*_vt(i)\). A foreign producer of variety \(j \neq i\) sells \(\tilde{q}^*_t(j)\) units in the foreign market at a price \(p^*_vt(j)\) and exports to the domestic market \(\tilde{q}_t(j)\) units at a price \(p_{bt}(j)\). The aggregate demand for a domestic variety \(i\) and an imported variety \(j\) both sold in the domestic market is given by:

\[
q_t(i) = P^*_t Q^*_t \cdot p^*_vt(i)^{-\sigma} , \quad \tilde{q}_t(j) = P^*_t Q^*_t \cdot p^*_vt(j)^{-\sigma} \tag{1}
\]

The aggregate demand in the foreign market is \(q^*_t(i) = P^*_t Q^*_t \cdot p^*_vt(i)^{-\sigma}\) and \(\tilde{q}^*_t(j) = P^*_t Q^*_t \cdot p^*_vt(j)^{-\sigma}\).

In what follows I consider the case of symmetric countries, such that \(N^* = N\), \(P^*_t = P_t\), \(Q^*_t = Q_t\), and I outline the equilibrium for the domestic economy only.

**Technology.** Production employs labor according to a linear technology parameterized by labor productivity \(a\). There exists a distribution of technological choices \(T(a)\) bounded over the real line \(a > a_{\min} > 0\). Firms are heterogeneous in terms of labor productivity, every firm is endowed with one productivity level and produces one and only one variety. The production function of a domestic firm endowed with productivity \(a\) is:

\[
y(a, l_t) = al_t \tag{2}
\]

where \(l_t\) is employment at time \(t\) and \(y(a, l_t)\) is output.

Exports are associated with an additional ad-valorem cost. In order to sell one unit of good in the export market a firm ships \(\tau \geq 1\) units. Let the indicator function \(1(a) = \{0, 1\}\) denote the exporter status: \(1(a) = 1\) indicates that the firm is an exporter, otherwise \(1(a) = 0\). Market clearing at the firm level and the feasibility of production imply:

\[
q_t(a) + 1(a) \tau q^*_t(a) = y(a, l_t) \tag{3}
\]

where I anticipate that the export decision is ultimately a function of firm productivity. Notice that since serving an export market requires \(\tau q^*_t\) units of additional production, if an exporter firm
demands \( l_t^d \) employees to serve the domestic market then additional \( l_t^e = \tau^{1-\sigma}l_t^d \) employees are demanded to serve the export market; which yields a total employment of \( l_t = (1 + 1(a)\tau^{1-\sigma}) l_t^d \).

**Revenue and firm behavior.** Firms maximize profit in each destination, based on consumer demand (1), technology (2) and market clearing (3). Equating marginal revenues across markets yields: \( p_{qt}^* = \tau p_{qt} \). The difference in prices translates into differences in demand in the two destination markets: \( q_t^* = \tau^{-\sigma}q_t \). Inverse demand and market clearing yield the firm’s total revenue as a function of productivity and employment:

\[
r(a, l_t) = \left[(1 + 1(a)\tau^{1-\sigma}) P_t^a Q_t^a \right]^\frac{1}{2} \left( al_t \right)^{\frac{\sigma-1}{\sigma}}
\]

Notice that total revenue can also be written as \( r(a, l_t) = (1 + 1(a)\tau^{1-\sigma}) r_t^d(a, l_t) \) where \( r_t^d(a, l_t) = (P_t^a Q_t^a)^\frac{1}{2} \left( al_t^{\sigma-1} \right)^{\frac{1}{\sigma}} \) is the revenue from domestic sales.

**Wage.** The labor market is not perfect, because of search and matching frictions. Firms employ many workers and when making their wage offer they anticipate the outcome of a bargaining with the marginal worker, as discussed in Stole and Zwiebel (1996). In the case that the two parties break the negotiation, the firm loses the profit from the marginal worker in the current period \( \partial r(a, l_t)/\partial l_t - w(a, l_t) \), and the worker loses the opportunity to earn a wage \( w(a, l_t) \). Assuming symmetric bargaining power between the two parties, the wage is obtained as the particular solution to the ordinary differential equation which equates the two option values:

\[
w(a, l_t) = \frac{\sigma - 1}{2\sigma - 1} \frac{r(a, l_t)}{l_t}
\]

which I will refer to as the wage equation.

### 2.1 Industry

Potential entrants pay a fixed cost \( f_e > 0 \) to enter the market in the following period. This investment covers the cost of hiring workers the firm will start matched with. Incumbent firms pay a fixed cost \( f_p > 0 \) each period to enable production. Firms that decide to export pay a fixed cost for operating in the export market \( f_x > 0 \). All fixed costs are in nominal terms.\(^7\)

Because of fixed costs, firms might not break-even and make losses. For this reason, at the beginning of every period \( t \) incumbents decide to compete or exit. Exit provides zero value. Hence, all firms that are sufficiently productive to realize a positive lifetime value remain, others choose optimally to exit; I refer to this choice as endogenous exit. Let the productivity cutoff \( a_t^{\text{in}} \) be the maximum threshold below which firms endowed with productivity \( a < a_t^{\text{in}} \) exit at time \( t \). In addition to endogenous exit, firms can be forced to exit because of an idiosyncratic destructive shock which occurs with exogenous probability \( \delta_f \in (0, 1) \); I describe such an event as exogenous exit.

Payments occur at the end of a period and total profit is then allocated to finance the entry of new firms. The mass of incumbent firms in a given period \( M_{t+1} \) consists of all previous incumbents

\(^7\)This assumption differs from Melitz (2003), where firms employ additional workers as a fixed factor of production. In the context of this paper, more productive firms will pay higher wages and a characterization of fixed costs in terms of employment would have made firms heterogeneous in terms of their fixed costs structure.
endowed with a productivity that is not lower than the current cutoff and did not exit because of an exogenous shock, plus new entrants which paid the entry cost in the previous period $E_t$ and realize a productivity that is not lower than the current cutoff:

$$M_{t+1} = \left[ 1 - T \left( a_{t+1}^{in} \right) \right] E_t + (1 - \delta_f) \mu(a_{t+1}^{in}; a_t^{in}) M_t$$

Equation (6) is the law of motion for the mass of firms in the industry, where $\mu(a; a_t^{in}) = \frac{1 - T(a)}{1 - T(a_t^{in})}$ for $a \geq a_t^{in}$ and 1 otherwise.

2.2 Labor market

Search is costly for firms and time consuming for workers: firms pay a cost $k > 0$ for each vacancy posted, unemployed workers send 1 job application per period, employed workers search by sending $\phi \in (0, 1]$ job applications per period. Matching is random and takes one period: vacancies posted in period $t - 1$ are matched with job applications sent in period $t - 1$, leading to job reallocation in period $t$. The matching technology is assumed to be homogeneous of degree 1 for vacancies and job applications, as documented in the empirical literature. In particular, I assume the matching technology proposed by Ramey et al. (2000) which has the advantage of a matching probability bounded in the unit interval for both workers and firms, one being the complement of the other. Hence, I call $x_t \in (0, 1)$ the probability that a worker matches with a firm in period $t$, while the probability that a vacancy is visited by a worker is $1 - x_t$. In this context, I will refer to the labor market in time $t$ as tighter when the probability that a worker receives a wage offer is higher.

The state of the labor market consists of the number of unemployed workers $u_t$ and the job finding probability $x_t$; which yields the state vector $z_t = \{u_t, x_t\}$. Two cumulative density functions (c.d.f.) characterize the labor market allocation conditional on the state of the labor market: $G(w; z_t)$ is the share of employed workers who accept a wage lower than $w$ or equal; $F(w; z_t)$ is the share of wage offers which will be accepted by a worker earning a wage lower than $w$ or equal.

Timing. At the end of every given period $t - 1$ there are $u_{t-1}$ unemployed workers and $u_{t-1} = N - u_{t-1}$ workers are employed. At the beginning of period $t$ the productivity cutoff $a_t^{in}$ is understood, and this determines firm exit and firm entry decisions. When a firm exits its jobs are destroyed. Hence, a share $\delta_f$ of jobs is destroyed because of exogenous firm exit; and a share $\epsilon_t \in [0, 1)$ of jobs is destroyed because of endogenous firm exit (the variable $\epsilon_t$ is an equilibrium outcome to be determined). In addition, jobs at firms which do not exit can be destroyed by an exogenous shock which occurs with probability $\delta_j \in (0, 1)$. Job creation consists of the vacancies posted in the previous period by both new firms which made a successful entry and previous incumbents which did not exit.

Following firm entry, exit and job destruction, the labor market opens. Vacancies and job applications are matched. Firms make offers simultaneously, without recall and, since discrimination between identical workers is not allowed, they offer the same wage to current employees and workers they match with on the market. Workers are either unemployed or they have in hand the wage offer from the employer to which they have been matched in the previous period. Workers who receive an offer on the market compare it with their current status and decide whether to accept the offer or
Worker reallocation. Workers are infinitely living and participate in the labor market to the end of maximizing their lifetime discounted income. A reservation wage for unemployed workers exists such that the value of being employed at this reservation wage makes workers indifferent with respect to remain unemployed. It can then be shown that rational unemployed workers accept every wage which is not lower than this reservation wage. For employed workers, rejecting an offer by employers met on the market does not lead to unemployment as they remain matched with their current employer. Moreover, because of costs associated with employer–to–employer reallocations, employed workers accept only offers which are better than the wage they currently earn. Changes in firm employment can be described in terms of the share of employed workers who separate from their current employer and the hiring success rate per vacancy posted.

A worker and a firm which offers a wage \( w \) separate because of an exogenous job destruction shock, or because the worker accepts a better job offer. The same probability of separation applies to all firm employees, and if thought of as a continuous measure then the share of employees who separate from a given firm which offers a wage \( w \) in the market at time \( t \) is:

\[
s(w; z_t) = \delta_j + (1 - \delta_j) \phi x_t [1 - F(w; z_t)]
\]  

(7)

which I will refer to as separation rate and where \( x_t [1 - F(w; z_t)] \) is the probability that a worker receives a wage offer better than \( w \). Firms match with a worker with probability \( 1 - x_t \) but a match becomes a new hiring if and only if the worker accepts the wage offer. The probability that a vacancy is filled for a firm which offers a wage \( w \) is given by the hiring rate

\[
h(w; z_t) = (1 - x_t) \left( \frac{u_{t-1} + \phi n_{t-1} (\delta + \varepsilon_t)}{u_{t-1} + \phi n_{t-1}} + \frac{\phi n_{t-1} (1 - \delta - \varepsilon_t) G(w; z_t)}{u_{t-1} + \phi n_{t-1}} \right)
\]  

(8)

where the total number of job applications is \( u_{t-1} + \phi n_{t-1} \), of which \( u_{t-1} + (\delta + \varepsilon_t) \phi n_{t-1} \) sent by workers who have unemployment as outside option, and \( (1 - \delta - \varepsilon_t) \phi n_{t-1} \) by workers who might remain matched with their current employer; then the parameter \( \delta = \delta_j + (1 - \delta_j) \delta_j \) accounts for the two causes of exogenous job destruction. The reallocation of workers occurs immediately after matching and determines the employment for every firm in the current period. The employment of a firm matched with \( l_{t-1} \) employees, which posted \( v_{t-1} \) vacancies and offers a wage \( w \) in the labor market at time \( t \), is given by:

\[
l_t = [1 - s(w; z_t)] l_{t-1} + h(w; z_t) v_{t-1}
\]  

(9)

which is implied by the balance between separations \( s(w; z_t) l_{t-1} \) and hirings \( h(w; z_t) v_{t-1} \). I will refer to equation (9) as the law of motion for employment.

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*The Bellman equations for the worker search problem are discussed in the appendix. The wedge between earned wages and accepted wages can be captured by modeling reallocation costs as a function of the earned wage. Under mild assumptions on the functional form, a correspondence between the two c.d.f. \( I(w; z_t) \) and \( G(w; z_t) \) exists and is unique. However, the reallocation of workers across firms can be fully discussed without further assumptions.
An incumbent firm makes \((1 - \delta_j) l_{t-1}\) job offers at a wage \(w\) to its previous employees whose jobs were not destroyed, and \((1 - x_t) \theta_{t-1}\) job offers to workers with whom the firm matches in the market. From a total of \((1 - \delta_j) l_{t-1} + (1 - x_t) \theta_{t-1}\) wage offers, the number of accepted job offers is given by the law of motion for employment (9). The employment of a new entrant, which is endowed with \(f_e/k\) vacancies financed with the entry cost, and offers the same wage \(w\), is \(h(w; z_t) f_e/k\). Since these two firms pay the same wage \(w\), and all workers receive offers from the market randomly, the acceptance rate per offer is the same for the two firms; which implies:

\[
\frac{1 - s(w; z_t)}{h(w; z_t)} = \frac{1 - \delta_j}{1 - x_t}
\]

Equation (10) establishes that the probability of hiring a worker who visited a firm vacancy is equal to the probability that a firm employee does not accept the job offer of another employer.

**Labor market dynamics.** The law of motion for unemployment is given by the balance between previously unemployed workers who do not find a job plus workers whose jobs are destroyed in the current period and do not receive job offers:

\[
u_{t+1} = (1 - x_{t+1}) u_t + (1 - \phi x_{t+1}) (\delta + \varepsilon_{t+1}) (N - u_t)
\]

Unemployment is a decreasing function of the current job finding probability and rises with endogenous job destruction. The system of separation rate (7), hiring rate (8) and the competition condition (10) evaluated at the lower bound of the wage support \(w_0\), such that \(F(w_0; x_t) = G(w_0; x_t) = 0\), allows the job finding probability to be written as:

\[
x_{t+1} = \frac{(1 - \delta - \varepsilon_{t+1}) (N - u_t)}{u_t + \phi (N - u_t)}
\]

Equation (12) yields the law of motion for job finding probability. The probability to find a job in a given period is decreasing in past unemployment and in the share of jobs destroyed in the current period. The system of difference equations (11) and (12) governs the dynamics of the labor market, given the share of jobs destroyed by endogenous exit, which is determined in general equilibrium as discussed in the following section. In steady state no endogenous firm exit occurs, since the cutoff productivity does not change by definition. It follows that steady state unemployment \(u = \frac{\phi \delta^2 N}{1 - 2\delta + \phi \delta^2}\) and job finding probability \(x = \frac{1 - 2\delta}{\phi (1 - \delta)}\) exist and are unique.\(^9\)

Out of the steady state, an unanticipated policy implementation causing endogenous firm exit also determines an endogenous job destruction. The variable \(a_t^{in} > a_t^{in} \implies \varepsilon_{t+1} > 0\) jumps to a strictly positive value at period of policy implementation. Unemployment jumps to a higher level than in the steady state \(u_t > u\) and job finding probability jumps to a lower level than in the steady state. The following dynamics toward the unique steady state is disciplined by the system (11)–(12) evaluated in the absence of further shocks \(\varepsilon_{t+1} = 0\). Substituting for \(x_{t+1}\) in the law of motion of unemployment (11) reveals that the dynamics of unemployment is autonomous and the future level

\(^9\)Notice that data on unemployment rate \(u/N\) and duration of unemployment spell \(1/x\) can be used to calibrate the parameters \(\delta\) and \(\phi\).
of unemployment is a strictly positive, increasing and convex function of the current level. Figure (1) shows the dynamic equation for unemployment. There is one non–trivial and stable steady state. The level of unemployment induced by endogenous firm exit is unstable. Starting from \( u_i \neq u \) the transition of unemployment toward its steady state is monotonic \( u_{t+1} \leq u_t \); as I briefly discuss in the appendix. Future level of job finding probability depends only on the current unemployment level. Hence, the dynamics of the job finding probability is monotonic and increasing \( x_{t+1} \geq x_t \).

Figure 1: Dynamics of the unemployment rate

**Firm vacancy posting.** The choice about the number of vacancies is made at the end of the period as the outcome of an inter-temporal optimality problem in which the firm maximizes its value. The endogenous aggregate state of the firm’s problem consists of job finding probability \( x_t \) and unemployment \( u_t \), while productivity \( a \) and firm employment \( l_t \) are the individual state variables. The number of vacancies \( v_t > 0 \) is the control variable. The value of the firm is given by:

\[
V(a, l_t; z_t) = \max_{v_t \geq 0} \left\{ r(a, l_t) - w(a, l_t) l_t - kv_t - f_p - 1(a) f_x + (1 - \delta_f) V(a, l_{t+1}; z_{t+1}) \right\}
\]

subject to the law of motion employment at the firm level (9), unemployment (11) and job finding probability (12); where the revenue satisfies (4), the wage equation (5) holds and the firm does not anticipate future policies causing endogenous exit.
3 Equilibrium of the labor market

In this section I define an equilibrium with wage dispersion. Differently from other models of the labor market which feature OJS, such as Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002), in this model heterogeneity in firm productivity implies wage dispersion and a monotonic increasing relationship between productivity, wage and employment. A second important distinction from most OJS frameworks is that this model allows for an explicit solution of the dynamics out of the steady state, in the absence of aggregate uncertainty. The framework is simpler than solution methods recently proposed, (such as Menzio and Shi (2010) for directed search or Moscarini and Postel-Vinay (2012) for random search), which accommodate aggregate uncertainty but do not allow for firm entry and exit.

DEFINITION. A monotone increasing wage dispersion equilibrium with firm entry consists of an assignment function \( \omega(a; z_t) : [a_1^m, \infty) \rightarrow [w_{0t}, w_{1t}] \) increasing in productivity and a hiring rate \( h(w; z_t) : [w_{0t}, w_{1t}] \rightarrow [h_{0t}, h_{1t}] \) that solve the firm problem (13) and such that the marginal firm which enters the market makes zero value.

A policy for vacancies \( \vartheta(a, l_t; z_t) \) which solves the firm problem (13) satisfies the necessary first order condition for an interior solution:

\[
(1 - \delta_f) h (w(a, l_{t+1}; z_{t+1}); z_{t+1}) \frac{\partial V (a, l_t; z_t)}{\partial l_t} = k
\]

where \( J(a, l_t; z_t) = \frac{\partial V (a, l_t; z_t)}{\partial l_t} \) is the value of the marginal job at time \( t \). The choice of an optimal number of vacancies is unconstrained due to the linear cost of vacancy posting. However, in a monotone increasing wage dispersion equilibrium with firm entry there is one feasible choice for the number of vacancies. In fact, if there exists a monotonic and increasing relationship between productivity and wage, then the wage equation (5) implies that every given level of productivity \( a \geq a_1^m \) maps into one and only one employment level \( l_t \). A new entrant endowed with productivity \( a \) at time \( t \) employs \( l_t = h (w(a, l_t); z_t) f_e/k \) workers and this has to be the employment of every firm endowed with the same productivity level; which also shows that more productive firms employ more workers because they pay higher wages. Substituting in the law of motion for employment (9) yields the policy for vacancies:

\[
\vartheta(a, l_t; z_t) = \left( \frac{1}{h (w(a, l_t); z_t)} - \lambda (u_t) \right) l_t
\]

where \( \lambda (u_t) = \frac{(1-\delta_f) [u_t + \phi(N-u_t)]}{\delta_f [1-\phi(N-u_t)]} \), decreasing in \( u_t \), is obtained after the substitution for competition in the labor market (10) and the law of motion of job finding probability (12) given that no endogenous firm exit is anticipated. Conditional on employment, firms post more vacancies the higher is the current level of unemployment. The profit of a firm endowed with productivity \( a \) at time \( t \) is:

\[
\pi(a, l_t; z_t) = \frac{\sigma}{\sigma-1} w(a, l_t) l_t + \lambda (u_t) k l_t - f_e - f_p - 1(a, l_t) f_p
\]

Hence, more productive firms make higher profits, for two reasons: they make a greater profit per worker and employ more workers. Substituting in the expression for the value of a firm
and differentiating with respect to employment yields the value of the marginal job \( J(a, l_t; z_t) = \frac{1}{\sigma} [w(a, l_t) + \lambda(u_t)k] \), which is simply the infinite sum of current marginal profit discounted by the firm destruction rate. The value of a firm endowed with productivity \( a \) at time \( t \) is: \( V(a, l_t; z_t) = \pi(a, l_t; z_t)/\delta f \). Given productivity and employment, any choice of vacancies \( \nu(a, l_t; z_t) \) should not affect the value of the firm, indeed it should imply the same profit. This property yields \( \frac{\partial \nu(a, l_t; z_t)}{\partial z_t} = \frac{w(a, l_t)}{k} \). Computing the partial derivative of vacancies with respect to current employment when the policy for vacancies is \( (15) \) yields a Bernoulli differential equation in \( h(w; z_t) \), which does not depend on productivity or employment. Since the wage–productivity assignment is increasing, the firm which pays the wage \( w_{0t} \) has the minimum hiring rate \( h_{0t} \). A solution that passes through the point \( h(w_{0t}; z_t) = h_{0t} \) exists, is unique and can be derived in closed form:

\[
 h(w; z_t) = \frac{h_{0t}}{\alpha(z_t) + \beta(z_t) \frac{w_{0t}}{w} - (\alpha(z_t) + \beta(z_t) - 1) \left( \frac{w}{w_{0t}} \right)^{\sigma}} \tag{17}
\]

where \( \alpha(z_t) = \lambda(u_t)h_{0t}, \beta(z_t) = \frac{\sigma - 1}{\sigma - 1} \frac{w_{0t}}{k} \). Evaluating the hiring rate \( (8) \) at the extremes of the wage support yields \( h_{0t} = (1 - x_t)(1 - \phi x_t) \) and \( h_{1t} = 1 - x_t \). Then for a given level \( w_{0t} \), the upper bound of the wage support is such that \( h(w_{1t}; z_t) = h_{1t} \). The c.d.f. of wages at which workers reallocate \( G(w; z_t) \) is obtained by inverting the hiring rate \( (8) \) and substituting for \( h(w; z_t) \) as in the equilibrium condition \( (17) \). Then, the separation rate \( (7) \) and the condition for competition in the labor market \( (10) \) determine the c.d.f. of wage offers \( F(w; z_t) \).

Notice that the hiring rate, and indeed the separation rate and the c.d.f. of wage offers, do not depend either on firm productivity, neither on the cutoff productivity. A clear implication of this labor market structure is that the only channel through which firms compete on the labor market is wage. All firms face the same increasing labor supply curve, which is given by \( \omega(a, l_t; z_t) = \omega(a, l_t) \). The system of revenue \( (4) \) and wage equation \( (5) \) yields labor demand at the firm level. The clearing of labor demand and supply at the firm level determines the assignment of wage to productivity.\(^{10}\) An explicit solution can be derived for the inverse function \( a = \omega^{-1}(w; z_t) \), which solves:

\[
 (1 + 1(a)^{1-\sigma}) \left( \frac{a}{a_{in}} \right)^{\sigma-1} = \frac{h(w; z_t)}{h(w_{0t}; z_t)} \left( \frac{w}{w_{0t}} \right)^{\sigma} \tag{18}
\]

Substituting in \( (17) \) and rearranging to explicit for the labor payroll then substituting for \( \frac{\sigma}{\sigma-1} w(a, l_t)I_t \) in \( (16) \) yields the profit of a firm as a function of firm’s productivity and of the state vector. The

\(^{10}\)Let \( p_t \) and \( l_t \) be respectively the price on the domestic market and the employment required to serve the domestic market for a firm endowed with productivity \( a \geq a_{in} \), producing \( al_t^d \) units of output for the domestic market and making a domestic revenue \( r_t^d(a) \). Then the wage equation \( (5) \) implies \( p_t = \frac{2x_t}{\sigma - 1} \frac{w}{n} \), where \( w \) is the wage paid by the firm endowed with productivity \( a \). Let \( r_t^d \) be the revenue on the domestic market made by a firm endowed with productivity \( a_{in} \) then the relative productivity satisfies \( \frac{a}{a_{in}} = \frac{w}{w_{0t}} \left( \frac{r_t^d(a)}{r_t^d(a_{in})} \right)^{\frac{1}{\sigma-1}} = \frac{w}{w_{0t}} \left( \frac{1}{1 + (1(\sigma - 1) \frac{r_t^d(a)}{r_t^d(a_{in})})^{\frac{1}{\sigma-1}}} \right) \).
maximum productivity level below which the value of a firm is negative solves \( \pi(a_t; z_t) = 0 \); and if there is selection in the export market then a firm endowed with the productivity cutoff does not export. These two conditions allow profit of a firm to be written as

\[
\pi(a; a_{in}^t) = \left[ (1 + 1(a)\tau^{1-\sigma}) \left( \frac{a}{a_{in}^t} \right)^{\sigma-1} - 1 \right] f_p - 1(a)f_x \tag{19}
\]

and determine the lower bound of the wage support

\[
w_{0t} = \frac{\sigma - 1}{\sigma} \left( \frac{f_p + f_e}{(1-x_t)(1-\phi x_t)f_e} - \lambda(u_t) \right) k \tag{20}
\]

Notice that the profit of a firm does not depend on the state of the labor market. This implies that firm entry, exit and export decisions can be evaluated independently on the transitional dynamics of the labor market. In fact, firm profit in equilibrium (19) has the same expression as the one in Melitz (2003). This might sound at the same time reassuring (since the setup of the industry is identical) and surprising (as this model features a wage dispersion equilibrium with unemployment).

The structure of the labor market has an effect on revenue, wage, employment and hiring costs, but these contributions cancel out in the expression of firm profit. As a consequence, the aggregation properties of the industry follow unchanged from the Melitz’s model.

Nevertheless, this model accounts for the endogenous job destruction due to trade induced selection and for the consequent transitional adjustment of the labor market. These two distinctive features have important implications not only for the labor market outcomes, but also for the industry dynamics. The analysis of endogenous job destruction concludes the equilibrium of the labor market; while aggregation and the termination of the general equilibrium are presented in the following section.

### 4 Industry equilibrium

In this section I describe the determination of the equilibrium, in two steps. First, export participation and the aggregate properties of the model are discussed, by looking at a representative firm which has employment equal to the average employment per firm in the economy. Then the model is closed and a distinction between the long–run and the short–run equilibrium is discussed.

#### 4.1 Aggregation

The profit of a firm in the domestic market is \( \pi^d(a; a_{in}^t) = [(a/a_{in}^t)^{\sigma-1} - 1] f_p \), while the profit in the export market is \( \pi^e(a; a_{in}^t) = (a/a_{in}^t)^{\sigma-1} \tau^{1-\sigma} f_p - f_x \). Since the value of a firm is proportional to firm profit, the indifference condition \( \pi^e(a_{x}^t; a_{in}^t) = 0 \) implies \( a_{x}^t = \tau(f_x/f_p)\pi^{\frac{1}{\sigma-1}} a_{in}^t \) which determines the productivity cutoff above which value maximizing firms select into the export market.

The revenue is a linear function of profit and employment.\(^{11}\) It follows that a firm which makes the average revenue \( \bar{r}_t = R_t/M_t \), also makes the average profit \( \bar{\pi}_t \) and employs the average number of workers \( (N - u_t)/M_t \). Let \( \bar{a}_t \) be the productivity of the firm with average employment, such

\(^{11}\) Using the wage equation (5) and the profit (19) to determine firm revenue yields \( r(a; l_t; z_t) = \frac{2}{\sigma-1} \left[ \pi(a; a_{in}^t) + f_p + 1(a)f_x + f_e - \lambda(u_t)k l_t \right] \). Details are reported in the appendix.
that $h(\omega(\tilde{a}_t; z_t); z_t) f_e/k = (N - u_t)/M_t$ is the firm employment and $\tilde{\omega}_t = \omega(\tilde{a}_t; z_t)$ is the firm wage. Moreover notice that total labor income in the economy is $L_t = \tilde{w}_t(N - u_t)$, hence $\tilde{w}_t$ solves $L_t = M_t \times \tilde{w}_t h(\tilde{w}_t; z_t) f_e/k$. Therefore, $\tilde{w}_t$ is the average wage across employed workers. The identity $r(\tilde{a}_t; z_t) = \bar{r}_t$ jointly with the expression for profit (19), and the wage assignment (18), implies the two equalities:

$$
(1 + 1(\hat{a}_t)^{-1}) (\bar{a}_t)_{a_t}^{\sigma-1} = \frac{h(\bar{w}_t; \tilde{z}_t)}{h(w_{0t}; \tilde{z}_t)} (\bar{w}_t/w_{0t})^{\sigma} = 1 + \frac{\pi_t + \mu_t f_e}{f_p}
$$

The condition (21) allows the average productivity $\bar{a}_t$ and the average wage $\bar{w}_t$ to be determined, given the state of the labor market $z_t = \{u_t, x_t\}$ and the pair of cutoff productivity and average profit $\{a_t^{in}, \bar{\pi}_t\}$.

Aggregate profit is used to finance the possible entry of new firms in the following period $f_t E_t = \bar{\pi}_t M_t$. Substituting in the the law of motion (6) yields the mass of firms $M_t$ as proportional to the entry probability

$$
M_t = \left[1 - T(a_t^{in})\right] \left(\bar{\pi}_{t-1} \epsilon_e + \frac{1 - \delta f}{1 - T(a_t^{in})}\right) M_{t-1}
$$

where the factor of proportionality is predetermined. A second expression for the mass of firms is implied by the labor market allocation. The equilibrium hiring rate (17) yields the average employment per firm $h(\bar{w}_t; z_t) f_e/k$. Therefore, the mass of firms satisfies the definition of average employment $M_t = (N - u_t)/(h(\bar{w}_t; z_t) f_e/k)$, which links the industry dynamics and the labor market dynamics.\(^{12}\)

Closing the model requires to determine six elements: a pair of values for the cutoff productivity and the average profit $\{a_t^{in}, \bar{\pi}_t\}$; the state vector of the labor market, which consists of unemployment and job finding probability $z_t = \{u_t, x_t\}$; then, the equilibrium conditions (21) and (22) together with the definition of average employment are sufficient to determine the average wage and the mass of firms $\{\bar{w}_t, M_t\}$. I summarize the short-run impact of trade looking at three points in time: $B$ indicates the steady state before a trade liberalization, $i$ indicates the time of implementation of a trade liberalization policy, $A$ indicates the steady state after a trade liberalization.

### 4.2 Endogenous job destruction

An increase of the productivity cutoff causes endogenous firm exit which leads to job destruction. Previous incumbents endowed with a productivity which is lower then the new cutoff exit the market and their employees become unemployed, before the labor market opens and production starts in the upcoming period. Assume $a_t^{in+1} > a_t^{in}$, then the number of firms that exit the market at the beginning of period $t + 1$ is given by $M_t^{exit} = [T(a_t^{in+1}) - T(a_t^{in})]/[1 - T(a_t^{in})] M_t$. The monotonicity of the wage–productivity assignment implies that the workers who lose their jobs because of endogenous firm exit

\(^{12}\)Closing the model through the economy budget constraint yields an equivalent determination of the mass of firms. The policy for vacancies (15) implies a total cost of vacancy posting $K_t = f_e M_t - k \lambda (u_t)(N - u_t)$. Total fixed costs for production and export are $(f_e + \mu_t f_e) M_t$. Total profit is $\Pi_t = \bar{\pi}_t M_t$, the number of potential new entrants in the next period is financed with current profit, such that: $f_e E_t = \Pi_t$. The budget constraint of the domestic economy is $R_t - f_t E_t = L_t + K_t + (f_p + \mu_t f_e) M_t$. Substituting for the aggregate variables yields the mass of firms as implied by the definition of average employment.
are the subset of the employed workers in the previous period who were earning a wage \( \omega(a_{t+1}^{in}; z_t) \) or lower. The c.d.f. of the distribution of wages across firms in a given period is a transformation of the exogenous productivity c.d.f. \( T(w; z_t) = T(\omega^{-1}(w; z_t)) \) and its density is defined by the wage productivity assignment (18). The hiring rate (17) determines firm employment, which is a continuous and increasing function of the wage. At the end of period \( t \) the number of workers employed in these firms which exit the market at time \( t + 1 \) is given by

\[
n_{t+1}^{ex} = M_{t+1}^{ex}h_{t+1}^{ex}f_e/k,
\]

where

\[
h_{t+1}^{ex} = \int_{w_{0t}}^{\omega(a_{t+1}^{in}; z_t)} h(w; z_t) \frac{dT(w; z_t)}{T(w_{0t}; z_t)}
\]
is the average hiring rate among the firms that exit at time \( t + 1 \), hence \( h_{t+1}^{ex}f_e/k \) is the average employment. The share of jobs destroyed because of endogenous firm exit is:

\[
\varepsilon_{t+1} = \frac{n_{t+1}^{ex}}{N - u_t}
\]

where \( \varepsilon_{t+1} > 0 \) if and only if \( a_{t+1}^{in} > a_{t}^{in} \), and \( \varepsilon_{t+1} = 0 \) otherwise. Condition (23) shows that for a given increase in the productivity cutoff, the larger is the share of workers employed at lower quantiles of the wage support the more severe the job destruction hits.

### 4.3 Long–run equilibrium

The long–run is a state of the economy in which: (i) the entry of firms is unbounded; (ii) the productivity cutoff and the mass of firms are in steady state; (iii) the labor market is in steady state. An unbounded entry of firms implies that the average profit of an incumbent firm is equal to the expected profit conditional on entry. Defining the function \( \varphi \) and its cutoff values

\[
\varphi(a') = \left[ \int_{a'}^{\infty} a'^{-1} \frac{dT(a)}{1 - T(a')} \right] \frac{1}{\sigma - 1}, \quad \varphi^m = \varphi(a_{t}^{in}), \quad \varphi^x = \varphi(a_{t}^{x})
\]

allows the conditional expected profit in the two markets to be written as \( \bar{\pi}_t^d = \bar{\pi}_t^d(\varphi^m; a_{t}^{in}) \) and \( \bar{\pi}_t^x = \bar{\pi}_t^x(\varphi^x; a_{t}^{in}) \). The expected profit of an incumbent firm is \( \bar{\pi}(a_{t}^{in}) = \bar{\pi}_t^d + \mu_t \bar{\pi}_t^x \) where \( \mu_t = \frac{1 - T(a_{t}^{in})}{1 - T(a_{t}^{in})} \) is the share of exporter firms. The export indifference condition implies that ceteris paribus the expected profit is higher the lower the barriers to trade. Accounting for this comparative statics, the expected profit conditional on entry is a function of the cutoff productivity \( \bar{\pi}(a_{t}^{in}; \tau, f_x) \) decreasing in the policy parameters \( \tau \) and \( f_x \). Let \( \bar{\pi}_s \) be the average profit evaluated in steady state \( t = s \), either before or after the trade liberalization. Then the zero profit condition

\[
\bar{\pi}_s = \bar{\pi}(a_{s}^{in}; \tau, f_x) \quad \text{for } s = B, A
\]
yields the the equality between average profit of an incumbent firm and expected profit conditional on entry. The expected value of entry (ex-ante the realization of firm productivity) is the expected value conditional on entry \( \bar{\pi}(a_{t}^{in}; \tau, f_x)/\delta_f \) times the probability of making a successful entry \( [1 - T(a_{t}^{in})] \).

---

13 The assignment (21) defines the inverse function \( a = \omega^{-1}(w; z_t) \), which is continuous and continuously differentiable over the support \( a_{t}^{in} \leq a < a_{t}^{x} \). Over this range of productivity values, the derivative \( da(w; z_t)/dw \) is positive, continuous and it can be computed analytically.
free entry, the expected value of entry cannot exceed the cost of entry. Therefore, in steady state a
_free entry condition_ holds:

\[
\frac{\pi(a_{s}^{in}, \tau, f_{x})}{\delta f} = \frac{f_{e}}{1 - T(a_{s}^{in})} \quad \text{for } s = B, A
\]  

(25)

A pair of values for the productivity cutoff and the average profit \(\{a_{s}^{in}, \pi_{s}\}\) which solves (24)–(25) exists and is unique. The state vector of the labor market has a unique steady state \(z_{s} = \{u_{s}, x_{s}\}\) which is determined by the labor market parameters only, as discussed in the third section. The equilibrium condition (21) determines a unique average wage \(\bar{w}_{s}\) for a given average profit \(\bar{\pi}_{s}\) and state of the labor market \(z_{s}\). Average employment \(h(\bar{w}_{s}; z_{s}) f_{e}/k\) is fixed by the equilibrium hiring rate (17). Total employment \(N - u_{s}\) is fixed by the unemployment level. Hence, there exists a unique value for the mass of firms \(M_{s} = (N - u_{s})/[h(\bar{w}_{s}; z_{s}) f_{e}/k]\) which satisfies the definition of average employment. This completes the determination of the steady state long–run equilibrium.

The steady state values of unemployment and job find probability depends only on the parameters of the labor market: size of the workforce \(N\), exogenous job destruction \(\delta\) and job applications by employed workers \(\phi\); as it is clear from the labor market dynamics (11)–(12). Comparative statics on the system (24)–(25) implies that the cutoff productivity \(a_{A}^{in} > a_{B}^{in}\) and the average profit \(\bar{\pi}_{A} > \bar{\pi}_{B}\) are higher in the steady state after a trade liberalization, while participating in the export market is easier \(a_{A}^{x} < a_{B}^{x}\). Under mild assumptions on the exogenous productivity distribution, also the sum \(\bar{\pi}_{i} + \mu_{i} f_{x}\) is necessarily higher in the steady state after the trade liberalization. And this is a sufficient condition for two results. First, after a trade liberalization there are less firms producing in the domestic market \(M_{A} < M_{B}\) but they employ on average more workers, as implied by the definition of average employment. Second, the average wage is higher after the trade liberalization \(\bar{w}_{A} > \bar{w}_{B}\), as implied by the equilibrium condition (21) given \(u_{A} = u_{B}\) and \(x_{A} = x_{B}\). This completes the steady state comparison of the economy before and after a trade liberalization.

4.4 Short–run equilibrium

The short–run is a state of the economy in which: (i) the long run patterns of firm entry, exit and export participation are correctly foreseen, but (ii) the current mass of firms, the average profit and the labor market allocation are not in steady state.

The time of implementation of a trade liberalization \(t = i\) is the first period of a short run equilibrium. Forward looking entry, exit and export decisions determine a more severe selection of incumbents and entrants and an increase in export participation:

\[
a_{i}^{in} = a_{A}^{in} > a_{B}^{in} \quad \text{and} \quad a_{i}^{x} = a_{A}^{x} < a_{B}^{x}.
\]

These patterns are suddenly updated at the implementation and then they are fixed for all the future periods \(t > i\). Therefore, a trade liberalization causes endogenous job destruction (23) at the time of implementation \(\varepsilon_{i} > 0\), but not later on \(\varepsilon_{t} = 0\) for every \(t > i\). This shock takes the labor market (11)–(12) out of its steady state, with higher unemployment and lower job finding probability:

\[
u_{i} > u_{A} = u_{B} \quad \text{and} \quad x_{i} > x_{A} = x_{B}.
\]

\footnote{Notice that the free entry condition can also be obtained as the unique steady state solution for the dynamics of the mass of firms (22).}
The following transitional dynamics of the labor market does not affect the productivity cutoffs neither the export participation, but still the adjustment of the industry equilibrium is not immediate. To see this is enough to evaluate the dynamics of the mass of firms (22), which yields to conclude that at the time of implementation the number of firms drops:

$$M_i = [1 - T(a_i^{in})] \left( \frac{\bar{\pi}_B}{f_e} + \frac{1 - \delta_i}{1 - T(a_i^{in})} \right) M_B < M_A < M_B.$$  

The number of possible entrants financed before the trade liberalization hits is not sufficient to compensate for the larger exit induced by the policy; and this explains $M_i < M_B$. Given the mass of firms at the time of implementation and the state of the labor market $z_i = \{u_i, x_i\}$, the average wage is determined by the definition of average employment $h(\bar{w}_i; z_i) f_e/k = (N - u_i)/M_i$.

The monotonicity of the hiring rate is sufficient to conclude that the average wage $\bar{w}_i$ exists and is unique, and it can be shown that the average wage at the time of implementation is lower than before the trade liberalization:

$$\bar{w}_i < \bar{w}_B < \bar{w}_A.$$  

The reason for this result is that the lower job finding probability $x_i < x_B$ decreases the pressure on wages; the proof is in the appendix. Two effects should be taken into account. First, the lower job finding probability decreases the value of unemployment during the bargaining. Second, the lower job finding probability decreases the risk that an employed worker receives a wage offer from a better firm. While the first mechanism is common to most search and matching environments, the latter channel is specific to a framework with OJS and it delivers the key implication that the lower the job finding probability the relative more workers are employed in relatively worse firms.

The intuition that at the time of implementation the relatively worse firms are relatively too big $h(\bar{w}_i; z_i)/h_{0i} < h(\bar{w}_B; z_B)/h_{0B}$ and do not suffer enough poaching from better firms $\bar{w}_i/<w_{0i} < \bar{w}_B/w_{0B}$ is also informative about the average profit, through the aggregate equilibrium condition (21). Therefore, the average profit of a domestic firm at the time of implementation is lower than before the trade liberalization

$$\bar{\pi}_i < \bar{\pi}_B < \bar{\pi}_A,$$

despite the average productivity is higher (because of a left truncation on the previous distribution of productivity). Hence, the reason for this result must be found in the allocation of workers as argued. This completes the determination of the short–run equilibrium, while a more detailed analysis of the impact of trade on worker reallocation is discussed the next section.

### 4.5 Welfare

In the Melitz’ model trade induced selection implies a higher productivity cutoff and this increases aggregate welfare due to efficiency gains. This argument is also working in the present model, but it is not the only mechanism through which trade leads to welfare gains and losses. In this model unemployment responds to a trade liberalization and firms pay different wages. These two contributions offer novel implications for the effect of a trade liberalization on aggregate welfare.

As in Melitz (2003) the welfare analysis can be based on a firm which is representative of the domestic market in the sense that firm’s revenue is equal to average sales per variety. Consider a firm
which serves a domestic demand $\bar{q}_t$ making the average revenue per variety $\bar{p}_t \bar{q}_t = \frac{R_t}{(1 + \mu_t)} M_t = \frac{\bar{r}_t}{1 + \mu_t}$. Substituting in the demand function (1), with $R_t = P_t Q_t$, allows the consumption based price index and the indirect utility from consumption to be written as: $P_t = [(1 + \mu_t)M_t]^{-\sigma \bar{p}_t} \bar{p}_t$ and $Q_t = [(1 + \mu_t)M_t]^{-\sigma \bar{q}_t} \bar{q}_t$. It can be shown that the quantity sold in the domestic market is proportional to the demand served by the cutoff firm $q_{0t} = a_{it} l_{0t}$, where $l_{0t} = h_{0t} f_e / k$ is the employment of the cutoff firm. Substituting in the expression for aggregate indirect utility yields welfare:

$$Q_t = (N - u_t)^{\sigma - 1} l_{0t}^{1 - \frac{1}{\sigma - 1}} (\bar{w}_t / w_{0t})^{\sigma - 1} a_{it}^{\sigma}$$  \(26\)

Aggregate welfare is the outcome of four multiplicative components: total employment $(N - u_t)$, employment at the cutoff firm $l_{0t}$, the ratio of average over cutoff wage $(\bar{w}_t / w_{0t})$ and the productivity cutoff $a_{it}^{\sigma}$. While the latter is the only determinant of welfare in the Melit’z model, in this paper the labor market equilibrium makes the welfare analysis richer. Unemployment $u_t \neq 0$, hiring frictions $h_{0t} \neq 1$ and wage dispersion $\bar{w}_t \neq w_{0t}$ contribute to welfare. Moreover, the trade induced change in the productivity cutoff is suddenly achieved at the time of implementation and it is permanent. Instead, unemployment, job finding probability and the wage ratio evolve over time while the economy follows a transitional dynamics toward the new steady state. Therefore, aggregate welfare follows a transitional dynamics because of the labor market adjustment.

### 4.6 Inequality

The coefficient of variation is a measure of income inequality, defined as the ratio of standard deviation over mean of the income distribution. Using the range of the wage support as a proxy for the variance, the coefficient of variation is approximated by:

$$cv_t \approx \sqrt{w_{1t} - w_{0t}} / w_t$$  \(27\)

Two channels affect inequality, measured as an increase in the coefficient of variation. Inequality increases the wider is the wage support and the lower is the average wage. It is possible to show that: (i) at the implementation, the wage support shrinks while in the long–run it converges to the same length as the steady state before the trade liberalization, (ii) the average wage is lower at the implementation than before the trade liberalization, but in the long–run the average wage reaches a higher steady state values than before the trade liberalization; see the appendix for the details. Although the two contributions contrast each other a clear pattern emerges. Inequality is higher at the time of implementation than before the trade liberalization. However, in the long–run the increase in average wage dominates and in the new steady state inequality is lower than before the trade liberalization.

### 5 Conclusion

This paper has analyzed the impact of trade on the labor market, describing the transitional dynamics of unemployment and probability to find a job in response to trade liberalization. Three strands of empirical findings motivates this study. First, trade induces firm exit which leads to a
peak in unemployment at the time of trade liberalization episodes. While the efficiency gains from increased aggregate productivity are observed, the recovery is sluggish and the job losses are offset only in the long run. Second, job–to–job reallocations are a main component of labor market adjustment in response to trade. Third, there is strong evidence on the importance of employment status (as opposed to unemployment) for explaining wage determination and worker reallocation. Under these premises, the effect of trade on the labor market can only be studied in a dynamic framework and allowing employed workers to search for better paid jobs, which explains time and direction of inter–firm reallocation.

The model features an industry equilibrium with heterogeneous firms, making forward looking decisions on entry, exit, and export participation; following Melitz (2003). In the labor market, on–the–job search (OJS henceforth) generates a wage dispersion equilibrium, along the lines of Mortensen (2010). This channel triggers competition among firms in the labor market, which explains both wage dispersion and job–to–job reallocations. Trade induces a selection of least productive firms out of the market, causing a sudden destruction of jobs. More productive firms pay better wage and are also the one which expand in the export market. Exporters increase their employment by poaching workers from other firms. Non–exporters (which still are productive enough to stay in the market) shrink, losing workers who move to exporter firms and partially replacing them by hiring from the pool of unemployed. Firm exit and the increase in average profit, due to trade, create favorable conditions for the entry of new firms in the future periods. The probability of finding a job falls in the short run because of firm exit, but then increases over the transition following exporter expansion and firm entry. Only in the long run the recovery of the labor market is complete. The steady state levels of unemployment and job finding probability are not affected by trade liberalization. Nevertheless, the analysis of the dynamics shows that a trade liberalization generates an equilibrium of the labor market in which unemployment is higher and job finding probability is lower than before the policy implementation.

The model remains highly tractable and has an analytical closed form solution. Remarkably, the industry equilibrium is as simple as Melitz (2003), but it allows for wage dispersion, unemployment and a fully dynamic analysis. I achieve this by making two modifications. First, I assume that vacancies and job applications issued in a period are matched in the following period. This disciplines the dynamics of the job finding probability. Second, I assume that the entry cost paid by a potential new entrant finances the vacancies the firm starts with, in case of successful entry. This determines employment in the first period, then the balance between separations and hirings disciplines firm employment over time. The simplicity of the model solution comes with some concessions. In common with all models with Dixit–Stiglitz preferences and linear production functions, firm revenue is a log–linear function of employment. This feature delivers a wage distribution that is independent of idiosyncratic firm productivity. Another concession is that there is no aggregate uncertainty. Conditional on the current policy environment, firms correctly anticipate the aggregate state of the economy when making their forward looking decisions.

The model shows that a trade liberalization generates rich transitional dynamics. In contrast, comparative statics between the two steady states are dramatically less informative. Moreover, this paper makes a point showing how the trade induced increase of efficiency comes at the cost of
worsening the opportunity of employment of workers searching for a job. This scenario might suggest a policy intervention which reduces the time of adjustment while not hindering the reallocation process. In my view, the model I propose in this paper provides a useful tool to design policies that balance the short–run costs and the long–run gains from a trade liberalization.
6 Appendix

6.1 Labor market equilibrium

In this section I present the matching technology which is consistent with the labor market equilibrium discussed in the paper and I present the Bellman equations for the value of employment and unemployment which prescribe the worker policies. I also show how the wedge between the c.d.f. of earned wages $I(w; z_t)$ and the c.d.f. of accepted wages $G(w; z_t)$ can be rationalized by taking into account reallocation costs. In the second part of this paragraph, I discuss the solution of the Bernoulli equation which yields the equilibrium hiring rate $h(w; z_t)$. Finally I analyze the dynamics of unemployment and job finding probability.

Matching technology. Each unemployed worker sent one application in the previous period, whereas each employed worker sent $\phi > 0$ applications. Firms posted $V_{t-1}$ vacancies, but because of exogenous and endogenous exit only a fraction $\rho_t$ of these is still open at time $t$. Therefore, every period $t$, after firm entry and exit occur, a mass of $\rho_t V_{t-1}$ vacancies is matched with $u_{t-1} + \phi n_{t-1}$ job applications. Matches are formed according to a technology which combines current vacancies $\rho_t V_{t-1}$ and job applications $u_{t-1} + \phi n_{t-1}$. The number of matches is given by:

$$M_t = \frac{\rho_t V_{t-1} (u_{t-1} + \phi n_{t-1})}{\rho_t V_{t-1} + (u_{t-1} + \phi n_{t-1})}$$

This functional form of matching technology was proposed by Ramey et al. (2000) and has the advantage of a matching probability bounded in the unit interval for both workers and firms, while the technology is homogeneous of degree 1 for vacancies and job applications, as documented in the empirical literature. Labor market tightness $\theta_t$ and the probability that a worker receives a job offer $x_t$ are:

$$\theta_t = \frac{\rho_t V_{t-1}}{u_{t-1} + \phi n_{t-1}}$$

$$x_t = (1 + 1/\theta_t)^{-1}$$

while the probability that a vacancy is visited by a worker is: $x_t/\theta_t = 1 - x_t$. Notice that the matching function implies that the number of workers who receive a wage offer $x_t(u_{t-1} + \phi n_{t-1})$ is equal to the number of visited vacancies $(1 - x_t)\rho_t V_{t-1}$. Imposing this functional form for the matching function allows the ratio of open vacancies in the market $\rho_t$ to be determined given the total number of vacancies posted in the previous period $V_{t-1}$. In the paper I focus on the determination of employment flows and job finding probability, vacancy turnover instead does not play any particular role. This is the reason why in the main text I abstract from a particular form of the matching process, without loss of generality for the goals of the paper.

Value of unemployment and value of employment. Let $U(z_t)$ be the value of being unemployed and $W(a; z_t)$ be the value of being employed in a firm with productivity $a$ when the state of the economy is $z_t$; then let $F(a; z_t) = F(\omega(a; z_t); z_t)$ be the distribution of wage offers over the productivity support $a \geq a^m$, as implied by the wage–productivity assignment (18). Conditional on receiving a wage offer in the next period, the expected value from searching for a worker who starts period $t$ employed in a firm with productivity $a$ is $S(a; z_{t+1})$. 

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In order to take into account reallocation costs associated with employer–to–employer flows, I define the continuous function \( m(a; z_t) \) such that \( m(a; z_t) \geq a \) for \( a \geq a_t^{in} \) and \( m(a; z_t) = a \) at \( a = a_t^{in} \) and \( a \to \infty \). Workers matched with employers endowed with productivity \( a \) accept the wage offer of a firm endowed with productivity \( a' \) if and only if \( W (a'; z_{t+1}) \geq W (m(a; z_t); z_{t+1}) \). Given this formulation, the value from searching is:

\[
S(a; z_{t+1}) = \int_{a_t^{in}}^\infty \max \{ W(a'; z_{t+1}), W(m(a; z_t); z_{t+1}) \} \, dF(a; z_t)
\]

\[
= W(a; z_{t+1}) + G(a; z_{t+1})
\]

where

\[
G(a; z_{t+1}) = [1 - F(m(a; z_t); z_{t+1})] \int_{m(a; z_t)}^{\infty} \left[ W(a'; z_{t+1}) - W(a; z_{t+1}) \right] \frac{dF(a'; z_{t+1})}{1 - F(m(a; z_t); z_{t+1})}
\]

is the (ex–ante) expected gain of voluntary quitting the job at a firm with productivity \( a \) for a firm with productivity higher or equal \( m(a; z_t) \). Notice that \( G(a; z_{t+1}) \geq 0 \) and the better the wage paid by the current employer the lower the margins to improve \( \frac{dG(a; z_{t+1})}{da} \leq 0 \). Furthermore, notice that the value from searching for a worker who is employed at the marginal producer \( S(a_t^{in}; z_{t+1}) \) is equal to the average value of being employed.

Unemployed workers do not earn labor income in the current period, they send one job application and they receive a job offer in the following period with probability \( x_{t+1} \). The value of searching for an unemployed worker (who does not have the option to remain matched with a current employer) is equal to the value of searching starting from the marginal producer \( S(a_t^{in}; z_{t+1}) \). The value of being unemployed at time \( t \) is:

\[
U(z_t) = x_{t+1} S(a_t^{in}; z_{t+1}) + (1 - x_{t+1}) U(z_{t+1})
\]

Employed workers in a firm endowed with productivity \( a \) earn a wage \( \omega(a; z_t) \), send \( \psi \in [0, \phi] \) job applications and suffer a loss of value \( \psi \epsilon \) if they search; where the parameter \( \epsilon \geq 0 \) indicates the effort cost of applying for a job. In the next period, employed workers might become unemployed because of an exogenous shock hits the firm or the job, with arrival rate \( \delta \). Otherwise the worker receives a renewal of the contract with the current employer. Regardless of the new employment status, an employed worker who sends \( \psi \) applications in the current period receives a wage offer in the future period with probability \( \psi x_{t+1} \). The value of employment is:

\[
W(a; z_t) = \omega(a; z_t) - \psi \epsilon + (1 - \delta) \left[ (1 - \psi x_{t+1}) W(a; z_{t+1}) + \psi x_{t+1} S(a; z_{t+1}) \right] + \delta \left[ (1 - \psi x_{t+1}) U(z_{t+1}) + \psi x_{t+1} S(a_t^{in}; z_{t+1}) \right]
\]

where \( \omega(a; z_t) \) is the continuous function mapping productivity into wage and, as discussed in the paper, the wage is a strictly positive increasing function of productivity. Since the gains from a voluntary reallocation are not increasing in the productivity of the current employer, the value of employment \( W(a; z_t) \) is increasing in the productivity of the current employer \( a \).
Worker policies. Comparing the value of being employed and sending ψ applications with the value of being employed and sending zero applications yields the difference between the benefit from searching ψx_{t+1}[(1−δ) (S(a;z_{t+1}) − W(a;z_{t+1})) + δ (S(a_{t+1}^m;z_{t+1}) − U(z_{t+1}))] which is a decreasing function of the productivity of the current employer, and the cost of searching which is constant ψε. Therefore, if employees of firms at the top of the productivity distribution search then all employed workers search. The benefit of searching for a → ∞ converges to δψx_{t+1} [S(a_{t+1}^m;z_{t+1}) − U(z_{t+1})]; that is the expected value of avoiding unemployment in case the current job is destroyed. I discuss the equilibrium of the labor market when workers with the best employment status are indifferent between searching or not δx_{t+1} [S(a_{t+1}^m;z_{t+1}) − U(z_{t+1})] = ϵ and I assume they search. Therefore all employed workers search, and they send the maximum number of applications ψ = 0, regardless the effort cost. Substituting this indifference condition in the value of employment and unemployment yields the surplus of being employed:

\[
W(a;z_t) − U(z_t) = ω(a;z_t) − ϵ/δ + (1−δ) [W(a;z_{t+1}) − U(z_{t+1})] + 
\]

\[
(1−δ) ϕx_{t+1} [S(a;z_{t+1}) − W(a;z_{t+1})]
\]

where ω(a;z_t) > ϵ/δ for every t is a sufficient condition for W(a;z_t) > U(z_t). A reservation wage w_R exists such that W(a;z_t) = U(z_t) for ω(a;z_t) = w_R. Let S_R(z_{t+1}) ≥ S(a_{t+1}^m;z_{t+1}) and W_R(z_{t+1}) ≥ U(z_{t+1}) be the value of searching and being employed starting from an employment at the reservation wage; then it can be shown by substitution that the reservation wage is decreasing in both S_R(z_{t+1}) and W_R(z_{t+1}). Hence, the maximum reservation wage w_R(z_t) is obtained when W_R(z_{t+1}) = U_{t+1} and S_R(z_{t+1}) = S(a_{t+1}^m;z_{t+1}), which yields:

\[
w_R = \frac{ε}{δ} − (1−δ) φx_{t+1} [S(a_{t+1}^m;z_{t+1}) − U(z_{t+1})]
\]

\[
= [1 − φ(1−δ)]ε/δ
\]

Notice that the maximum reservation wage is constant over time, equal to the effort cost when employed workers send as many applications as unemployed workers φ = 1, lower (greater) than the effort cost if employed workers search more (less) than unemployed workers. The policy of unemployed workers is established: for a lower bound of the wage distribution w_{0t} > w_R unemployed workers accept every wage offer.

The surplus of being employed in a firm with productivity a' instead of a is:

\[
W(a';z_t) − W(a;z_t) = ω(a';z_t) − ω(a;z_t) + (1−δ) [W(a';z_{t+1}) − W(a;z_{t+1})] + 
\]

\[
(1−δ) φx_{t+1} [G(a';z_{t+1}) − G(a;z_{t+1})]
\]

Workers employed in a firm endowed with productivity a who received an offer by a firm with productivity a' accept and reallocate if and only if W(a';z_t) ≥ W(a;z_t). By definition, if there are costs associated with employer–to–employer reallocation then a' ≥ a is not a sufficient condition for reallocation. Instead, there exists a minimum offer ω(m(a;z_t);z_t) accepted by workers employed at a wage ω(a;z_t). The value ω(m(a;z_t);z_t) can then be interpreted as the reservation wage for poaching workers employed at a wage ω(a;z_t).

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15 This feature is common to models with OJS, it reflects the the different probability of finding a job when starting in the two employment status.
Let \( C(\omega(a; z_t); z_t) \geq 0 \) be the lifetime loss of value due to reallocation for a worker earning \( \omega(a; z_t) \). This includes a continuation value which balances the losses \( W(a; z_{t+1}) + \phi x_{t+1} G(a; z_{t+1}) \) and the gains \( W(m(a; z_t); z_{t+1}) + \phi x_{t+1} G(m(a; z_t); z_{t+1}) \) from entering the future labor market in the new employment status, such that the current value of the reallocation cost is:

\[
\begin{align*}
  c(\omega(a; z_t); z_t) &= C(\omega(a; z_t); z_t) + \\
  &- (1 - \delta) [W(m(a; z_t); z_{t+1}) - W(a; z_{t+1})] + \\
  &- (1 - \delta) \phi x_{t+1} [G(m(a; z_t); z_{t+1}) - G(a; z_{t+1})]
\end{align*}
\]

Then the reservation wage \( \omega(m(a; z_t); z_t) \) for poaching a worker employed at wage \( \omega(a; z_t) \) satisfies

\[
W(m(a; z_t); z_t) = W(a; z_t) + C(\omega(a; z_t); z_t),
\]

which implies:

\[
\omega(m(a; z_t); z_t) = \omega(a; z_t) + c(\omega(a; z_t); z_t)
\]

and \( c(\omega(a; z_t); z_t) \geq 0 \). The policy for employed workers can now be determined looking at \( \omega(m(a; z_t); z_t) \) as a reservation wage: a worker employed in a firm with productivity \( a \) at a wage \( \omega(a; z_t) \) accepts wage offers from firms with productivity \( m(a; z_t) \) or higher, indeed for a wage that is \( \omega(m(a; z_t); z_t) \) or greater.

Reallocation costs determine a wedge between the earned wage and the wage at which an employed worker is willing to reallocate. Modeling the current value of the reallocation cost \( c(\omega(a; z_t); z_t) \) is out of the scope of this paper. Nevertheless, if the current value of the reallocation cost can be thought as continuous function such that higher earned wage implies higher reservation wage

\[
\omega(a; z_t) \geq \omega(\tilde{a}; z_t) \implies \omega(m(a; z_t); z_t) \geq \omega(m(\tilde{a}; z_t); z_t)
\]

then the following two results hold without loss of generality. First, the c.d.f. of accepted wages shows stochastic dominance of the first order on the c.d.f. of earned wages, such that \( G(w; z_t) \leq I(w; z_t) \) where the equality is strict in the interior of the wage support. Second, conditional on the state of the labor market \( z_t \), there exists a continuous and increasing function \( \ell(w; z_t) : [w_{0t}, w_{1t}] \rightarrow [w_{0t}, w_{1t}] \), where \( \ell(w; z_t) \leq w \), \( \ell(w_{0t}; z_t) = w_{0t} \) and \( \ell(w_{1t}; z_t) = w_{1t} \), such that

\[
I(\ell(w; z_t); z_t) = G(w; z_t) \iff I(w; z_t) = G(\ell^{-1}(w; z_t); z_t)
\]

for every \( w \in [w_{0t}, w_{1t}] \), and

\[
\tilde{w}_t = w_{0t} + \int_{w_{0t}}^{w_{1t}} [1 - G(\ell^{-1}(w'; z_t); z_t)] \, dw'
\]

since the implied average wage across workers is an equilibrium outcome of the model. Therefore, modeling the reallocation cost \( c(\omega(a; z_t); z_t) \) is equivalent to specify the function \( w' = \ell(w; z_t) \leq w \) where \( w \) is the lowest wage for which workers who are currently earning \( w' \) accept to reallocate. The function \( \ell(w; z_t) \) must be continuous, increasing and bounded within the wage support and satisfy the restriction imposed on the implied average wage. As a simple example consider:

\[
\ell(w; \rho_t) = w_{0t} + (w_{1t} - w_{0t}) \left( \frac{w - w_{0t}}{w_{1t} - w_{0t}} \right)^{\rho_t}
\]

\[
\rho_t : \tilde{w}_t = w_{0t} + \int_{w_{0t}}^{w_{1t}} [1 - G(\ell^{-1}(w'; \rho_t); z_t)] \, dw'
\]

where \( \ell(w; \rho_t) < w \) for at least a \( w \in (0, 1) \) implies \( \rho_t > 1 \) and the monotonicity of \( \ell(w; \rho_t) \) with respect to \( \rho_t \) implies existence and uniqueness of \( \rho_t \) for every given average wage \( \tilde{w}_t \). 23
Derivation of the equilibrium value of a job. Consider the value of the marginal job that solves the firm inter–temporal problem (13):

$$J(a, l_t; z_t) = \frac{\partial r(a, l_t; z_t)}{\partial l_t} - \frac{\partial w(a, l_t; z_t)}{\partial l_t}l_t - w(a, l_t) - k\vartheta(a, l_t; z_t) + (1 - \delta_f) \frac{\partial l_{t+1}}{\partial l_t}J(a, l_{t+1}; z_{t+1})$$

The wage equation (5) implies \(\frac{\partial w(a, l_t; z_t)}{\partial l_t}l_t = w(a, l_t)\). After the substitution of the policy for vacancy (15),

$$\vartheta(a, l_t; z_t) = \frac{l_t}{h(w(a, l_t); z_t)} - \lambda(u_t)l_t$$

in the law of motion for employment (9), future employment is proportional to current employment

$$l_{t+1} = \frac{h(w(a, l_{t+1}; z_{t+1}))}{h(w(a, l_t); z_t)}l_t,$$

since \(\lambda(u_t) = \frac{1 - s(w; z_t)}{h(w; z_t)}\) when the competition in the labor market (10) is understood, then after the substitution of \(x_{t+1}\) as implied by the law of motion (12), given \(\varepsilon_{t+1} = 0\).

Equivalently the next period employment can be written as follows

$$l_{t+1} = (\vartheta(a, l_t; z_t) + \lambda(u_t)l_t) h(w(a, l_{t+1}); z_{t+1})$$

hence \(\frac{\partial l_{t+1}}{\partial l_t} = \left(\frac{\partial \vartheta(a, l_t; z_t)}{\partial l_t} + \lambda(u_t)\right) h(w(a, l_{t+1}); z_{t+1})\). Substituting in the value of a job yields

$$J(a, l_t; z_t) = w(a, l_t) - k\frac{\partial \vartheta(a, l_t; z_t)}{\partial l_t} + \left(\frac{\partial \vartheta(a, l_t; z_t)}{\partial l_t} + \lambda(u_t)\right) k$$

where \((1 - \delta_f) h(w(a, l_{t+1}); z_{t+1})J(a, l_{t+1}; z_{t+1}) = k\) as implied by the first order condition for vacancy posting (14).

Derivation of the equilibrium hiring rate. If the wage is an increasing function of productivity and the employment of a firm which pays a wage \(w\) satisfies \(l_t = h(w; z_t)\frac{t}{k}\) then employment is an increasing function of productivity. Moreover, although the monotone wage–productivity assignment can be weakly increasing, the employment–wage assignment is strictly increasing for \(w \in [w_{0t}, w_{1t}]\), as implied by the c.d.f. \(G(w; z_t)\). Hence, given firm productivity \(a\) and the aggregate state vector \(z_t\), firm employment is fixed, and the value of the marginal job, indeed the value of a firm, are determined. This implies that the profit \(\pi(a; z_t) = r(a, l_t) - w(a, l_t)l_t - k\vartheta(a, l_t; z_t)\) does not depend on the vacancies \(\vartheta(a, l_t; z_t)\) posted at time \(t\). Taking the partial derivative with respect to employment yields:

$$\frac{\partial \vartheta(a, l_t; z_t)}{\partial l_t} = \frac{1}{k} \left(\frac{\partial r(a, l_t; z_t)}{\partial l_t} - \frac{\partial w(a, l_t; z_t)}{\partial l_t}l_t - w(a, l_t)\right) = \frac{w(a, l_t)}{k}$$

where the second equality is implied by the wage equation (5). A second independent determination of the same partial derivative is obtained directly from the policy (15) which yields

$$\frac{\partial \vartheta(a, l_t; z_t)}{\partial l_t} = \frac{h(w(a, l_t); z_t) - \frac{h(w(a, l_t); z_t)}{\partial w} \frac{\partial w(a, l_t)}{\partial l_t} l_t}{h(w(a, l_t); z_t)^2} - \lambda(u_t)$$

$$= \frac{h(w(a, l_t); z_t) + \frac{\partial \vartheta(a, l_t; z_t, w)}{\partial w} w}{h(w(a, l_t); z_t)^2} - \lambda(u_t)$$

where the second equality is obtained after the substitution \(\frac{\partial w(a, l_t; z_t)}{\partial l_t} l_t = -\frac{w}{\sigma}\) as implied by the wage equation (5). The system of the two conditions yields:

$$\frac{h(w; z_t)}{\partial w} + \frac{w}{\sigma} h(w; z_t) = \left(\frac{\sigma}{k} + \frac{\sigma \lambda(u_t)}{w}\right) h(w; z_t)^2$$
which is the canonical form of a Bernoulli differential equation. The general solution is:

\[ h(w; z_t) = \frac{1}{\sigma - 1} w + \lambda(u_t) + w^\sigma c \]

where \( c \) is the constant of integration. The boundary condition \( h(w_0; z_t) = h_0 \) yields

\[ c = \left( 1 - \frac{\sigma}{\sigma - 1} \frac{h_0}{k} w_0 \right) - \left( \frac{\sigma}{\sigma - 1} \frac{h_0 \lambda(u_t)}{k} \right) \]

hence the unique particular solution passing through the boundary condition is

\[ h(w; z_t) = \frac{h_0}{\lambda(u_t)} + \frac{\sigma}{\sigma - 1} \frac{w_0 h_0}{k} \frac{w}{w_0} - \left( \frac{\sigma}{\sigma - 1} \frac{h_0 \lambda(u_t)}{k} \right) \left( \frac{w}{w_0} \right)^\sigma \]

which is (17) for \( \alpha(z_t) = \lambda(u_t)h_0, \beta(z_t) = \frac{\sigma}{\sigma - 1} \frac{w_0 h_0}{k} \).

**Analysis of the labor market dynamics.** Substituting for 1 − \( x_{t+1} = \frac{u_t + (\delta + \varepsilon_{t+1})(N - u_t)}{u_t + \phi(N - u_t)} \) and 1 − \( \phi x_{t+1} = \frac{u_t + (\delta + \varepsilon_{t+1})(N - u_t)}{u_t + \phi(N - u_t)} \) from (12) in (11) yields a quadratic equation in \( u_t \).

The system of difference equations which governs the dynamics of the labor market is:

\[ u_{t+1} = \frac{[u_t + (\delta + \varepsilon_{t+1})(N - u_t)]^2}{u_t + \phi(N - u_t)} - \frac{(1 - \phi)(N - u_t)u_t}{u_t + \phi(N - u_t)} \]

\[ x_{t+1} = \left( \frac{1}{\delta - \varepsilon_{t+1}} \right) \left( \frac{1}{1 - \delta} \right) \left( \frac{(1 - \delta)(N - u_t)}{u_t + \phi(N - u_t)} \right) \]

Notice that for \( u_t \to N \) then \( u_{t+1} = N \) and for \( u_t \to 0 \) then \( u_{t+1} = \frac{(\delta + \varepsilon_{t+1})^2}{\phi} N > 0 \). Let \( A = \delta + \varepsilon_{t+1} \in (0, 1) \), \( B = [(1 - A)^2 + (1 - \phi)] \) and \( D = [2A(1 - A) - (1 - \phi)] \) then

\[ u_{t+1} = \frac{Bu_t^2 + DNu_t + A^2 N^2}{(1 - \phi)u_t + \phi N} \]

\[ \frac{du_{t+1}}{du_t} = \frac{B(1 - \phi)u_t^2 + B2\phi Nu_t + (\phi D - (1 - \phi)A^2) N^2}{[(1 - \phi)u_t + \phi N]^2} \]

Notice that the minimum level of unemployment implied by the law of motion is \( \phi u_t = A^2 N \).

Substituting implies that for every \( \phi \in [\delta + \varepsilon_{t+1}, 1] \) if \( 2B > 1 - \phi \) then the numerator of the first order derivative is strictly positive. This sufficient condition is always satisfied as \( B > 1 - \phi \), therefore \( \frac{du_{t+1}}{du_t} > 0 \). In order to determine the convexity I organize the discussion in two cases. For \( \phi = 1 \) it is immediate to show that the second order derivative is positive. For \( \phi < 1 \) it is convenient to notice that the derivative of the numerator with respect to \( u_t \) is \( B/(1 - \phi) \) times the derivative of the denominator with respect to \( u_t \); and the denominator is increasing in \( u_t \). Therefore

\[ \frac{d^2u_{t+1}}{du_t^2} > 0 \iff \frac{B}{1 - \phi} [(1 - \phi)u_t + \phi N]^2 > B(1 - \phi)u_t^2 + B2\phi Nu_t + (\phi D - (1 - \phi)A^2) N^2 \]

\[ \iff B\phi^2 + \phi A^2(1 - \phi)^2 > (1 - \phi)\phi D \]

\[ \iff \frac{(1 - A)^2\phi}{1 - \phi} + \frac{A^2(1 - \phi)}{\phi} + 1 > 2A(1 - A) \]

which is always satisfies since \( A = \delta + \varepsilon_{t+1} \in (0, 1) \). This concludes the proof. The dynamic equation for \( u_{t+1} \) is a positive, increasing and convex function of \( u_t \). Since \( u_{t+1} \) attains a strictly positive value as \( u_t \to 0 \) the 45-degree line cuts from below, which implies the existence and uniqueness of one steady state and a stable trajectory.
6.2 Industry equilibrium

In this section I discuss the derivation of the profit function and the revenue function. Furthermore, I prove existence and uniqueness of the productivity cutoffs \( a^{in}_t \) and \( a^{in}_t \) and I prove the impact of a trade liberalization on the industry equilibrium: higher productivity cutoff \( \frac{d a^{in}_t}{d x} < 0 \), lower export cutoff \( \frac{d a^{in}_t}{d x} > 0 \) and higher average profit plus average fixed costs \( \frac{\bar{\pi} + f_t}{M_t} < 0 \).

**Derivation of the profit function.** Rearranging the expression for the hiring rate (17) to explicit for the ratio \( \frac{h(w,z)}{h_0} \left( \frac{w}{w_0} \right)^{\alpha} \) then substituting for \( \frac{h(w,z)}{h_0} \left( \frac{w}{w_0} \right)^{\alpha} = (1 + 1(a)^{\tau-1}) \left( \frac{a}{a^{in}_t} \right)^{\sigma-1} \) from (18) yields

\[
\left( \frac{\alpha(z_t)}{h_0} + \beta(z_t) \frac{w}{w_0} h_0 \right) h(w; z_t) = \left( 1 + (1 + 1(a)^{\tau-1}) (\alpha(z_t) + \beta(z_t) - 1) \left( \frac{a}{a^{in}_t} \right)^{\sigma-1} \right).
\]

Employment satisfies \( l_t = h(w; z_t) \frac{f_t}{K} \), which substituting in the previous expression allows labor payroll to be written

\[
w_l = \frac{w_0 h_0}{\beta(z_t) K} \left[ \left( 1 + (1 + 1(a)^{\tau-1}) (\alpha(z_t) + \beta(z_t) - 1) \left( \frac{a}{a^{in}_t} \right)^{\sigma-1} \right) f_e - \frac{\alpha(z_t) k}{h_0} l_t \right]
\]

Looking at the the profit function (16), which is \( \pi(a, l_t; z_t) = \frac{\sigma}{\sigma-1} w(a, l_t) l_t + \lambda(u_t) k l_t - f_e - f_p - 1(a, l_t) f_x \), it is then immediate to substitute back for \( \alpha(z_t) = \lambda(u_t) h_0 K \), \( \beta(z_t) = \frac{\sigma}{\sigma-1} \frac{w_0 h_0}{K} \) where it is convenient, such that labor payroll can be written as follows:

\[
w_l = \frac{\sigma-1}{\sigma} \left[ (1 + 1(a)^{\tau-1}) f_e (\alpha(z_t) + \beta(z_t) - 1) \left( \frac{a}{a^{in}_t} \right)^{\sigma-1} + f_e - \lambda(u_t) k l_t \right]
\]

Substituting in (16) yields the profit:

\[
\pi(a; z_t) = (1 + 1(a)^{\tau-1}) f_e (\alpha(z_t) + \beta(z_t) - 1) \left( \frac{a}{a^{in}_t} \right)^{\sigma-1} - f_p - 1(a) f_x .
\]

**Derivation of the revenue function.** The condition \( \pi(a^{in}_t; a^{in}_t) = 0 \) implies \( f_e (\alpha(z_t) + \beta(z_t) - 1) = f_p \), which substituting in the expression for labor payroll yields:

\[
w_l = \frac{\sigma-1}{\sigma} \left[ (1 + 1(a)^{\tau-1}) \left( \frac{a}{a^{in}_t} \right)^{\sigma-1} f_p + f_e - \lambda(u_t) k l_t \right]
\]

Recognizing the same functional form of the profit \( \pi(a; a^{in}_t) \) yields:

\[
w_l = \frac{\sigma-1}{\sigma} \left[ \pi(a; a^{in}_t) + f_p + 1(a) f_x + f_e - \lambda(u_t) k l_t \right]
\]

The wage equation (5) can now be used to explicit the revenue as a function of profit and employment:

\[
r(a, l_t; z_t) = \frac{2\sigma-1}{\sigma} \left[ \pi(a; a^{in}_t) + f_p + 1(a) f_x + f_e - \lambda(u_t) k l_t \right]
\]

The average revenue across firms is:

\[
\bar{r}_t = \frac{2\sigma-1}{\sigma} \left[ \bar{\pi}_t + f_p + \mu f_x + f_e - \lambda(u_t) \frac{(N - u_t)}{M_t} \right]
\]

where \( (N - u_t)/M_t \) is the average employment per firm by definition.
Existence and uniqueness of the productivity cutoffs, and the impact of trade. As claimed in the text, the proof of existence and uniqueness of a cutoff productivity $a^m_t$ is equivalent to the one in Melitz (2003). Let $\kappa(a') = \left( \frac{\varphi(a')}{a'} \right)^{\sigma-1} - 1 = \int_a^\infty \left( \frac{a}{a'} \right)^{\sigma-1} \frac{dT(a)}{T(a)} - 1 > 0$. An application of the Leibniz rule yields:

$$\frac{d\kappa(a')}{da'} = \frac{\kappa(a')dT(a')/da'}{1 - T(a')} - (\sigma - 1) \frac{1 + \kappa(a')}{a'} \frac{\partial f}{\partial a}$$

Define the function

$$j(a') = [1 - T(a')]\kappa(a') \geq 0$$

then the following properties hold:

$$\frac{dj(a')}{da'} = -\frac{1}{a'} (\sigma - 1)[1 - T(a')] [1 + \kappa(a')] \leq 0 \quad \text{and} \quad \frac{dj(a')}{da'} \frac{d}{j(a')} = -(\sigma - 1) \frac{1 + \kappa(a')}{\kappa(a')} < -(\sigma - 1).$$

The free entry condition (25) can be written as:

$$j(a^m_t)f_p + j(a^*_{t})f_x = \delta_ff_x$$

where $a^*_t = \tau(f_x/f_p)^{\frac{1}{1-\tau}}a^m_t$. Hence the left hand side is monotonically decreasing in $a^m_t$ on the positive real line $(0, \infty)$. The right hand side is constant and strictly positive, which implies existence and uniqueness of $a^m_t$. In open economy, keeping $a^m_t$ constant a change in $f_x$ implies a change in $j(a^*_t)$ as follows:

$$\frac{\partial j(a^*_t)}{\partial a^*_t} \frac{\partial a^*_t}{\partial f_x} f_x + j(a^*_t) = \left( \frac{\partial j(a^*_t)}{\partial a^*_t} \frac{\partial a^*_t}{\partial f_x} f_x + 1 \right) j(a^*_t) < 0$$

where the sign is implied by $\frac{\partial j(a^*_t)}{\partial f_x} f_x = \frac{1}{\sigma - 1}$ and the elasticity of the $j(a')$ function. The right hand side of the free entry condition is constant, therefore the new intersection identifies a new productivity cutoff $\frac{da^m_t}{df_x} < 0$. The change in $a^*_t$ is $\frac{da^*_t}{df_x} = \frac{a^*_t}{a^m_t} \frac{da^m_t}{df_x} + \frac{1}{\sigma - 1} a^*_t f_x$, therefore an evaluation of $\frac{da^m_t}{df_x}$ is necessary to determine the sign of $\frac{da^*_t}{df_x}$. The average profit can be written as:

$$\bar{\pi}_t = \kappa(a^m_t)f_p + \frac{1 - T(a^*_t)}{1 - T(a^m_t)} \kappa(a^*_t)f_x$$

Rewriting the free entry condition and taking the total derivative with respect to $f_x$ yields:

$$\frac{dj(a^m_t)}{da^m_t} \frac{da^m_t}{df_x} f_p = -\frac{dj(a^*_t)}{da^*_t} f_x - j(a^*_t)$$

$$\frac{dj(a^m_t)}{da^m_t} \frac{da^m_t}{df_x} f_p = -\frac{dj(a^*_t)}{da^*_t} \left( \frac{a^*_t}{a^m_t} \frac{da^m_t}{df_x} + \frac{1}{\sigma - 1} a^*_t f_x \right) f_x - j(a^*_t)$$

$$\frac{da^m_t}{df_x} = \frac{1 - T(a^*_t)}{\delta_ff_x} f_p + \frac{dj(a^*_t)}{da^*_t} \frac{a^*_t}{a^m_t} f_x < 0$$

where in the second line the change in $a^*_t$ is implied by the export indifference condition: $\frac{da^*_t}{df_x} = \frac{a^*_t}{a^m_t} \frac{da^m_t}{df_x} + \frac{1}{\sigma - 1} a^*_t f_x$ and the sign of the third line is implied by $dj(a')/da' < 0$. Rewriting the first line yields

$$\frac{da^*_t}{df_x} = \frac{-1}{\delta_ff_x} \left( \frac{dj(a^*_t)}{da^*_t} \frac{da^m_t}{df_x} f_p + j(a^*_t) \right) > 0$$
where the expression in the brackets is positive, hence the sign is determined.

The free entry condition allows $\bar{\pi}_t + \mu_t f_x$ to be written as

$$\bar{\pi}_t + \mu_t f_x = \frac{\delta_T f_x}{1 - T(a_t^x)} + \frac{1 - T(a_t^{in})}{1 - T(a_t^{in})} f_x$$

Hence the change in $\bar{\pi}_t + \mu_t f_x$ can be analyzed by studying the components

$$g(a) = \frac{a dT(a) / da}{T(a)} > 0$$

$$t(a_t^{in}, a_t^x) = \frac{[1 - T(a_t^x)] f_x}{1 - T(a_t^{in}) [1 + \kappa(a_t^{in})] f_x + [1 - T(a_t^x)] [1 + \kappa(a_t^x)] f_x} \in (0, 1)$$

$$\varepsilon_t^{in} = \frac{d a_t^{in}}{f_x} a_t^{in} = -\frac{t(a_t^{in}, a_t^x)}{\sigma - 1} < 0$$

$$\varepsilon_t^x = \frac{d a_t^x}{f_x} a_t^x = \frac{1 - t(a_t^{in}, a_t^x)}{\sigma - 1} > 0$$

Since the average profit increases following a trade liberalization, a sufficient (although not necessary) condition for higher $\bar{\pi}_t + \mu_t f_x$ is that $d(\mu_t f_x)/d f_x \leq 0$ which is true if and only if

$$-g(a_t^x)\varepsilon_t^x + g(a_t^{in})\varepsilon_t^{in} + 1 \leq 0,$$

which substituting for the elasticities simplifies to

$$(1 - t(a_t^{in}, a_t^x)) g(a_t^x) + t(a_t^{in}, a_t^x) g(a_t^{in}) \geq \sigma - 1.$$  

The left hand side is a convex combination of $g(a_t^{in})$ and $g(a_t^x)$, hence the condition is satisfied for $\min \{g(a_t^{in}), g(a_t^x)\} \geq \sigma - 1$. Assuming that $g(a)$ is an increasing function over the entire support $(0, \infty)$ yields the sufficient condition

$$g(a_t^{in}) \geq \sigma - 1 \implies \frac{d(\bar{\pi}_t + \mu_t f_x)}{d f_x} < 0.$$  

The regularity condition on $g(a)$ applies to a large class of distributions which are commonly used to fit the c.d.f. of total factor productivity across firms. And when such a property is met then the sufficient condition for $\frac{d(\bar{\pi}_t + \mu_t f_x)}{d f_x} < 0$ amounts to assume that the exogenous minimum value of productivity is sufficiently large compared to the elasticity of substitution across varieties.

The effect of a decrease in the variable trade cost $\tau$ on $a_t^{in}$ and $a_t^x$ follows the same approach than a decrease in the fixed cost. Furthermore, the comparative statics of a change in $\bar{\pi}_t$ and $\mu_t$ increase when the variable trade cost falls, while the fixed cost is unchanged.

### 6.3 Welfare

Let $\bar{a}_t^d$ be the productivity of the firm which charges a price on the domestic market $\bar{p}_t^d$ and serves a domestic demand $q_t^d$ making the average revenue per variety $\bar{p}_t^d q_t^d = \frac{R_t}{(1 + \mu_t) M_t} = \bar{r}_t = \bar{r}_t^d$. Substituting in the demand function (1), with $R_t = P_t Q_t$, allows the consumption based price index and the indirect utility from consumption to be written as: $P_t = [(1 + \mu_t) M_t]^{-\frac{1}{\sigma r_t}} \bar{r}_t^d$ and

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The price the representative firm charges on the domestic market is \( \bar{p}_t^d \). Let \( \bar{w}_t^d \) be the wage paid by a firm endowed with productivity \( \bar{a}_t^d \) then the productivity of the representative firm satisfies \( \left( \frac{\bar{a}_t^d}{a_t^d} \right)^{\sigma - 1} = \frac{\bar{r}_t}{r_t} \left( \frac{\bar{w}_t^d}{w_t^d} \right)^{\sigma - 1} \). Substituting for average employment and for (21) yields

\[
\bar{a}_t^d = \left( \frac{1}{(1 + \mu_t)M_t} \right)^{\frac{1}{\sigma - 1}} \left( \frac{N - u_t \bar{w}_t}{h_0 t f_e/k w_t} \right)^{\frac{1}{\sigma - 1}} \bar{w}_t^d a_t^d
\]

Substituting for average employment and for (21) yields

\[
\bar{w}_t^d = \left( \frac{1 + \mu_t}{1 + \bar{a}_t^d} \right)^{\frac{1}{\sigma - 1}} \bar{w}_t^d
\]

The price the representative firm charges on the domestic market is \( \bar{p}_t^d = \frac{2 \sigma - 1}{\sigma - 1} \bar{w}_t^d \) which implies that the quantity sold in the domestic market \( \bar{q}_t = \frac{\bar{r}_t / \bar{p}_t^d}{1 + \mu_t} \) is proportional to the demand served by the cutoff firm \( q_0 t = a_t^d h_0 t f_e/k \) as follows

\[
\bar{q}_t^d = \frac{N - u_t \bar{w}_t}{(1 + \mu_t)M_t} \bar{w}_t^d a_t^d = \left( \frac{1}{(1 + \mu_t)M_t} \right)^{\frac{1}{\sigma - 1}} \left( \frac{N - u_t \bar{w}_t}{h_0 t f_e/k w_t} \right)^{\frac{1}{\sigma - 1}} q_0 t
\]

Substituting in the expression for aggregate indirect utility yields welfare (26).

6.4 Discussion on the impact of trade

I discuss the impact of trade looking at three points in time: \( B \) indicates the steady state before a trade liberalization, \( i \) indicates the time of implementation of a trade liberalization policy, \( A \) indicates the steady state after a trade liberalization. Unemployment and job finding probability jump at the time of implementation and then converge to the same steady state level: \( u_i > u_B = u_A \) and \( x_i < x_B = x_A \). During the transition both variables follow a monotonic path: \( u_i > ... > u_t > u_{t+1} > ... > u_A \) and \( x_i < ... < x_t < x_{t+1} < ... < x_A \).

The equation for the cutoff wage (20) shows that the revenue of the cutoff firm, which is proportional to \( w_0 h_0 t \), is a decreasing function of the product \( \lambda(u_t) h_0 t \). The variable \( \lambda(u_t) \) is decreasing in unemployment, while \( h_0 t = (1 - x_t)(1 - \phi x_t) \) is decreasing in job finding probability, hence the interpretation of the change in the product \( \lambda(u_t) h_0 t \) is not immediate. However, substituting (10) and (12) in the definition of \( \lambda(u_t) \) yields \( \lambda(u_t) h_0 t = (1 - \delta_j) \left( \frac{1 - x_i}{1 - x_{i+1}} \right)^{(1 - \phi x_i)} \). Since \( \frac{1 - x_i}{1 - x_{i+1}} > 1 \) then \( \lambda(u_t) h_0 t > (1 - \delta_j)(1 - \phi x_i) > (1 - \delta_j)(1 - \phi x_B) = \lambda(u_B) h_0 B = \lambda(u_A) h_0 A \). Therefore,

\[
w_0 h_0 i < w_0 B h_0 B = w_0 A h_0 A
\]

and the dynamics of \( w_0 h_0 t \) exhibits the same increasing convergence of the job finding probability. The opposite is true for the product \( \lambda(u_t) h_0 t \). This result implies that the wage of the cutoff firm follows the same path of the job finding probability:

\[
w_0 i < w_0 B \quad \text{and} \quad w_0 i < ... < w_0 t < w_0 t+1 < ... < w_0 A = w_0 B
\]

\[
h_0 i > h_0 B \quad \text{and} \quad h_0 i > ... > h_0 t > h_0 t+1 > ... > h_0 A = h_0 B
\]

The implementation of a trade liberalization induces a selection of the small firms out of the market. Hiring decisions were predetermined, and starting from a steady state allocation they were simply
aimed to compensate exogenous job destruction which hits uniformly across the wage distribution. Therefore among the remaining firms, average employment per firm is higher and the employment level at the new cutoff is closer to the average employment than before:

\[
h(\bar{w}_i; z_i) > h(\bar{w}_B; z_B) \quad \text{and} \quad \frac{h(\bar{w}_i; z_i)}{h_{0i}} < \frac{h(\bar{w}_B; z_B)}{h_{0B}}.
\]

Substituting the cutoff wage (20) in the equation for the equilibrium hiring rate (17) shows that the employment ratio \(h(w; z_i)/h_{0i}\) is increasing in both the wage ratio \(w/w_0\) and the product \(\lambda(u_i)h_{0i}\). Since \(\lambda(u_i)h_{0i} > \lambda(u_B)h_{0B}\) then a lower employment ratio can be implied only by a lower wage ratio. Moreover since \(w_{0i} < w_{0B}\) then the average wage is also lower in absolute terms:

\[
\bar{w}_i < \bar{w}_B \quad \text{and} \quad \frac{\bar{w}_i}{w_{0i}} < \frac{\bar{w}_B}{w_{0B}}.
\]

As shown in (21), the average productivity ratio is fixed by

\[
\frac{h(\bar{w}_i; z_i)}{h_{0i}} \left(\frac{\bar{w}_0}{w_{0i}}\right)^\sigma < \frac{h(\bar{w}_B; z_B)}{h_{0B}} \left(\frac{\bar{w}_B}{w_{0B}}\right)^\sigma
\]

which determines the average profit gross of the cost for export participation \(\bar{\pi}_t + \mu_t f_x\). Given that the average cost of export participation does not decrease after the trade liberalization, the average profit of a domestic firm is lower at the time of implementation,

\[
\bar{\pi}_i < \bar{\pi}_B,
\]

despite the average productivity is higher (because of a left truncation on the previous distribution of productivity). Notice that this result implies that the free entry condition (25) cannot be satisfied at the time of implementation.

In the long run, unemployment, job finding probability and indeed the extremes of the wage support are equal to the initial steady state. Therefore, the equilibrium hiring rate yields the same map from wage to employment than before the trade liberalization but the higher productivity cutoff implies higher wage, by (21) which in turn yields higher average employment. Again the equilibrium condition (21) evaluated in the long run explains the higher average profit:

\[
h(\bar{w}_A; z_A) > h(\bar{w}_B; z_B), \quad \bar{w}_A > \bar{w}_B \quad \text{and} \quad \bar{\pi}_A > \bar{\pi}_B.
\]

The mass of incumbent firms at the time of implementation is given by

\[
M_i = \left[1 - T(\alpha^n_B)\right] \left(E_B + \frac{1 - \delta_f}{1 - T(\alpha^n_B) M_B}\right) \quad \text{where} \quad E_B = \frac{\delta f M_B}{1 - T(\alpha^n_B)}
\]

because it is the mass of entrants which would compensate for exogenous firm exit, starting from the initial steady state. Hence \(M_i = \frac{1 - T(\alpha^n_B)}{1 - T(\alpha^n_B) M_B} M_B < M_B\) and total profit is \(\bar{\pi}_i M_i < \bar{\pi}_B M_B\). Therefore the total number of potential entrants which can be financed at the time of implementation is less than before \(E_i < E_B\). Substituting for \([1 - T(\alpha^n_B)] = (M_B/E_B) \delta_f\) and \([1 - T(\alpha^n_A)] = (M_A/E_A) \delta_f\) in (6) yields

\[
\frac{M_i}{M_B} = \frac{M_A}{M_B} \left[\frac{E_i}{E_A} \delta_f + \frac{E_B}{E_A} (1 - \delta_f)\right] \quad \text{and} \quad \frac{M_{i+1}}{M_i} = \frac{M_A}{E_A} \frac{E_B}{M_i} \delta_f + (1 - \delta_f)
\]

The first equation is a sufficient condition for \(E_i < E_B \implies M_i < M_A\). The second equation is informative about the evolution of the mass of firms when it is interpreted jointly with the condition for financing entry, which implies

\[
\frac{E_i}{M_i} = \frac{\bar{\pi}_i}{f_e} < \frac{E_B}{M_B} = \frac{\bar{\pi}_B}{f_e} < \frac{E_A}{M_A} = \frac{\bar{\pi}_A}{f_e}.
\]
At the time of implementation the mass of firms is not in a stable point. And substituting for $E_i < \delta_i M_i/[1 - T(a_i^e)]$ in (6) shows that the next period mass of incumbent firms is even lower $M_{i+1} < M_i$. Given the same selection (the productivity cutoff is unchanged) the mass of incumbent firms is even lower than at the time of implementation. This boosts the average profit and indeed the value of entry from the following period on, above the level at which the free entry condition would be satisfied. This explains why the mass of firms increases converging to $M_A$ from below, as the share of entrants is above the one in steady state $E_t/M_t \geq E_A/M_A$ for every period after the implementation. Hence the average profit and the expected value of entry are larger than in the long run steady state during the entire transition.

**Change in welfare.** Notice that $h(\bar{w}_i; z_i) < h(\bar{w}_B; z_B)$ and $\bar{w}_{0i} < \bar{w}_B$ have several implications. First, through the wage equation, they imply $\bar{r}_i < \bar{r}_B$. Hence, since $w_0 h(w_0, z_i) < w_0 B h(w_0, z_B)$ and $M_i < M_B$ then average and aggregate profit fall

$$\bar{r}_i < \bar{r}_B \quad \text{and} \quad \bar{R}_i < \bar{R}_B.$$  

This also implies that the average revenue of the representative domestic firm is lower $\bar{r}_i^d < \bar{r}_B^d$, as $\mu_i > \mu_B$. To the end of understanding the change in welfare at the implementation, I use the definition of aggregate revenue $P_i Q_i = R_i$ and the definition of consumption based price index to obtain the price ratio:

$$\frac{P_B}{P_i} = \left( \frac{M_i}{M_B} \right)^{\frac{1}{\tau - 1}} \left( \frac{1 + (\bar{a}_i)^{\tau - 1}}{1 + (\bar{a}_B)^{\tau - 1}} \right)^{\frac{1}{\tau - 1}} \frac{\bar{w}_i}{\bar{w}_B} \frac{\bar{a}_i}{\bar{a}_B}$$

Then, the condition (21) allow to substitute for

$$\left( \frac{1 + (\bar{a}_i)^{\tau - 1}}{1 + (\bar{a}_B)^{\tau - 1}} \right)^{\frac{1}{\tau - 1}} \frac{\bar{a}_i}{\bar{a}_B} = \left( \frac{h(\bar{w}_i; z_i)/h_0}{h(\bar{w}_B; z_B)/h_0} \right)^{\frac{1}{\sigma - 1}} \left( \frac{\bar{w}_i/w_0}{\bar{w}_B/w_0} \right)^{\frac{\sigma}{\sigma - 1}} \frac{a_i^m}{a_B^m} < \frac{a_i^m}{a_B^m}$$

which implies:

$$\frac{P_B}{P_i} < \left( \frac{M_i}{M_B} \right)^{\frac{1}{\tau - 1}} \frac{\bar{w}_B a_i^m}{\bar{w}_i a_B^m} = \left( \frac{M_i}{M_B} \right)^{\frac{1}{\tau - 1}} \frac{p_0 B}{p_0 i} \implies \frac{P_B}{P_i} < \frac{p_0 B}{p_0 i}.$$  

Therefore, the change in welfare can be investigated by looking at $\frac{Q_i}{Q_B} = \frac{R_i}{R_B} \frac{P_B}{P_i}$, which yields

$$\frac{Q_i}{Q_B} < \frac{R_i}{R_B} \frac{p_0 B}{p_0 i} \left( \frac{R_i}{R_B} \right)^{1 - \frac{1}{\tau - 1}} \left( \frac{r_0 i}{r_0 B} \right)^{\frac{1}{\tau - 1}}$$

where the equality is obtained by substituting for $\frac{p_0 B}{p_0 i} = \left( \frac{R_B}{R_i} \right)^{\frac{1}{\tau - 1}} \left( \frac{r_0 i}{r_0 B} \right)^{\frac{1}{\tau - 1}}$ as implied by the demand equation (1). The wage equation (5) can then be used to rewrite the ratio in revenues

$$\frac{Q_i}{Q_B} < \left( \frac{R_i}{R_B} \right)^{\frac{1}{\tau - 1}} \left( \frac{w_0 h_0}{w_0 B h_0 B} \right)^{\frac{1}{\tau - 1}} = \left( \frac{\bar{w}_i (N - u_i)}{\bar{w}_B (N - u_B)} \right)^{\frac{1}{\tau - 1}} \left( \frac{w_0 h_0 i}{w_0 B h_0 B} \right)^{\frac{1}{\tau - 1}}.$$  

Since $w_0 h_0 i < w_0 B h_0 B$ and $R_i < R_B$ a sufficient condition for $\frac{Q_i}{Q_B} < 1$ is $\sigma \geq 2$, (although it is not a necessary condition).
Change in inequality. As already discussed, the relative employment $\frac{h(w; z_t)}{h_{0t}}$ is increasing in $w/w_{0t}$ and in $\lambda(u_t)h_{0t}$. Evaluating at the highest wage $w_{1t}$ the relative employment is given by $\frac{h_{1t}}{h_{0t}} = 1/(1-\phi x_t)$, hence it is an increasing function of $x_t$. It follows that the decrease in relative employment at the time of implementation $\frac{h_{1t}}{h_{0t}} < \frac{h_{1B}}{h_{0B}}$, when $\lambda(u_t)h_{0t} > \lambda(u_B)h_{0B}$, can only be explained by a lower wage ratio which also implies a lower upper bound of the wage support and a shorter wage support:

$$\frac{w_{1i}}{w_{0i}} < \frac{w_{1B}}{w_{0B}} \iff \frac{w_{1i}}{w_{1B}} < \frac{w_{0i}}{w_{0B}} < 1 \quad \text{and} \quad \frac{w_{1i} - w_{0i}}{w_{1B} - w_{0B}} < \frac{w_{0i}}{w_{0B}} < 1.$$ 

This shortening of the wage support points in the direction of a reduction of inequality, but this effect is contrasted by the change in average wage. The change $\frac{\sqrt{w_{1i} - w_{0i}}}{w_{1i}} - \frac{\sqrt{w_{1B} - w_{0B}}}{w_{1B}}$ is positive if and only if

$$\bar{w}_B > \bar{w}_i \left( \frac{w_{1B} - w_{0B}}{w_{1i} - w_{0i}} \right)^{1/2} > \bar{w}_i \left( \frac{w_{0B}}{w_{0i}} \right)^{1/2} \iff \frac{w_{1B}}{w_{0B}} > \frac{w_{0B}}{w_{0i}} \left( \frac{w_{0B}}{w_{1B}} \right)^{-1/2}$$

which is always satisfied, since $w_{0B} > w_{0i}$ and $\frac{\bar{w}_B}{\bar{w}_0} > \frac{\bar{w}_i}{\bar{w}_0}$.

6.5 Parameter restrictions

Two conditions must be checked. First, the hiring rate should be an increasing function of the wage for all $w \geq w_{0t}$. This boils down to check whether the denominator of 17 is a decreasing function of the wage ratio $w/w_{0t}$. This is true if and only if $\sigma(\alpha(z_t) + \beta(z_t) - 1)(w/w_{0t})^{\sigma-1} \geq \beta(z_t)$. Since $w \geq w_{0t}$ and $(\alpha(z_t) + \beta(z_t) - 1)f_p/f_e$ from the zero profit condition, then:

$$\beta(z_t) \leq \frac{\sigma f_p}{f_e} \iff \frac{dh(w; z_t)}{dw} \geq 0, \forall w \geq w_{0t}$$

Rewriting $\beta(z_t) = \frac{\sigma}{\sigma - 1} \frac{w_{0d}h_{0d}}{h_{0t}}$, and using the equation (20) for the minimum wage $w_{0t}$ where $h_{0t} = (1 - x_t)(1 - \phi x_t)$ yields the equivalent sufficient condition $\lambda(u_t)h_{0t} \geq 1 - (\sigma - 1) \frac{f_p}{f_e}$. Since $x_t < 1$ and $\phi \leq 1$ by construction such that $h_{0t} > 0$ and $\lambda(u_t) > 0$ then

$$f_p \geq \frac{f_e}{\sigma - 1}$$

is a sufficient (although not necessary) condition for the existence of a wage dispersion equilibrium.

The second condition which it must be fulfilled is that firms post a strictly positive number of vacancies. Notice that since $h(w; z_t) \geq h_{0t}$ for every $w \geq w_{0t}$ then if firms at the top of the wage distribution post a strictly positive number of vacancies this must be true for the less productive firms as well. An analysis of the policy for vacancies (15) shows that firms post a strictly positive number of vacancies if and only if $\lambda(u_t)h(w_{1t}; z_t) < 1$. A sufficient condition can be obtained when $u_t$ is at its minimum, so $\lambda(u_t)$ is at its maximum, and when $x_t$ is at its minimum, such that $h(w_{1t}; z_t) = 1 - x_t$ is at its maximum. Considering episodes of trade liberalization, the minimum level of unemployment is reached in steady state, hence for $x = \frac{1 - \delta}{\phi(1-\delta)}$ then $\lambda(u) = \frac{1 - \delta}{1 - \delta_j} = \frac{(1-\delta_i)\phi(1-\delta)}{\phi(1-\delta) - (1-\delta_j)}$. Let $x_{min}$ be the lower bound of job finding probability, such that $x \geq x_{min}$ then $\lambda(u_t)h(w_{1t}; z_t) \leq (1 - \delta_j) \frac{1 - x_{min}}{1 - x} < 1$ is satisfied when the probability of filling a vacancy is bounded above

$$1 - x_{min} < \frac{1 - x}{1 - \delta_j} = \frac{\phi(1-\delta) - (1 - 2\delta)}{(1 - \delta_j)\phi(1-\delta)}$$
that is a sufficient condition for all firms posting a strictly positive number of vacancies. However, a necessary condition would be substantially less demanding, since this sufficient condition is satisfied when unemployment is at its minimum (so in steady state) and also the job finding probability is at its minimum (so at the impact); which cannot happen simultaneously. Notice that the minimum of job finding probability is reached in the presence of job destruction, from (12). Therefore a lower bound on the job finding probability is equivalent to an upper bound for the endogenous job destruction rate:

\[
\varepsilon_{\text{max}} < \frac{\phi(1 - \delta_j) - (1 - 2\delta)(1 - \delta)\delta_j}{1 - 2\delta}
\]

Once more, notice that this upper bound is sufficient but it imposes a much stricter requirement than it is necessary to guarantee that all firms post a strict number of vacancies. However, this condition can be checked from (23) given the particular choice of a family for the wage distribution \(\varepsilon_{\text{max}} = I(w^*; z)\) evaluated at the initial steady state \(z = \{a_{i+1}^m, u, x\}\) for that particular wage \(w^* = \omega(a_{i+1}^m; z)\) which solves the wage productivity mapping (18) at the new productivity cutoff \(a_{i+1}^m\).

Vacancies are posted under the assumption that firms do not anticipate endogenous exit. Notice that \(\lambda(u_t) = 1 - \delta_j\) given \(\varepsilon_{t+1} = 0\). Therefore, the lower bound of the wage support is given by:

\[
w_{0t} = \frac{\sigma - 1}{\sigma} \left( \frac{f_p + f_e}{f_e} \frac{1}{h_{0t}} - \frac{1 - \delta_j}{1 - x_{t+1}} \right) k
\]

Taking the first order derivative of \(\frac{dw_{0t}}{dx_{t}}\) shows that its sign is determined by the algebraic sum

\[
\frac{(1-\delta_j)}{[1-x_{t+1}]^2} \frac{dx_{t+1}}{dx_t} - \frac{f_e}{f_e} \frac{2\phi x_t - (1 + \phi)}{[(1-x_t)(1-\phi x_t)]^2}, \text{ where } \frac{dx_{t+1}}{dx_t} \geq 0.
\]

Then

\[
x_t \leq \frac{1 + \phi}{2\phi} \quad \forall \phi \leq 1
\]

is a sufficient condition for \(\frac{dw_{0t}}{dx_{t}} \geq 0\). A necessary and sufficient condition for \(x < 1\) is \(\phi > \frac{1 - 2\delta}{1 - \delta}\), therefore the space for the searching effort of employed workers is:

\[
\frac{1 - 2\delta}{1 - \delta} < \phi \leq 1
\]

which is wider the higher is the total probability of exogenous job destruction \(\delta\). A necessary and sufficient condition for selection of firms in the export market is

\[
f_x > \tau^{-(\sigma - 1)} f_p
\]

such that \(a_x^e > a_{i+1}^m\). The two restrictions \(f_e > 0\) and \(k > 0\) guarantee a positive but finite number of vacancies, \(\delta_j > 0\) yields a positive unemployment rate, \(\delta_f > 0\) yields a stable mass of firms and \(\delta = \delta_f + \delta_j - \delta_f \delta_j < \frac{1}{2}\) guarantees a steady state unemployment rate which is less than 1.
References


