The Influence of Altruistic Preferences on the Race to the Bottom of Welfare States*

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Abstract

Common tax competition models suggest that in the absence of migration costs and other barriers to free migration, welfare states will undercut each other’s tax rate to attract taxpayers and keep welfare recipients at bay. This will lead to a zero-taxation outcome in the absence of migration costs or other barriers to migration. This paper shows that under the assumption of altruistically motivated taxpayers, tax competition leads to unique pure strategy Nash equilibria in taxation which are different from zero given sufficiently strong altruistic preferences. This result is robust to varying formulations of the altruistic preferences. It is also shown that if countries differ in the number of welfare recipients, unique pure strategy Nash equilibria exist in which the country with the fewer poor attracts more taxpayers and sets higher taxes. This translates into small countries setting higher tax rates than large ones, a result that stands in contrast to common theories of tax competition.

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1. Introduction

Globalization and European integration have exercised a significant influence on migration patterns for decades now. Not only the inhabitants of poor 2nd and 3rd world countries searching for a better life and more opportunities have shown a higher propensity to migrate, but also the population of the rich Western countries has become more mobile. About 100,000 Germans emigrated from Germany in 1991, a figure which has risen to roughly 160,000 in 2007 (Bundesamt für Statistik (2010)). Most migrants chose other European countries as their new homes. It has long been feared by policy makers and argued by economists that this migration between seemingly rather similar countries could be driven, at least in parts, by tax differentials. The “Sachverständigenrat Deutscher Stiftungen für Migration”\textsuperscript{2} published a report in 2009 admonishing that mainly highly qualified German citizens migrate and that the net-migration of German citizens is negative. Furthermore, they remark that qualitative losses in education and skills in the workforce are not at all compensated by unskilled immigration from non-OECD countries because Germany is among the most attractive countries for low-skilled immigrants.\textsuperscript{3} Having citizens leave the country after they have just finished their education is especially bad, as the costs for training will not be reimbursed by future tax payments (Holzner et al. (2009)). Most taxation, be it income tax, property tax, compulsory health or unemployment insurance, constitutes a form of wealth transfer from taxpayers in the higher income brackets (who are usually the well-educated) to those with low or no income.

For these reasons, traditional theory argues that countries compete for a mobile tax base consisting of redistribution-averse citizens which leads to a destructive Race to the Bottom in taxes, making the provision of tax financed public goods and welfare transfers all but impossible in the worst case.\textsuperscript{4} The empirical evidence for this phenomenon, as will be shown later, is rather mixed, however.

But also countries with high tax rates seem to have a high appeal for taxpaying migrants, as can be seen from European migration patterns of highly qualified personnel (see Figure 1).

\textsuperscript{2} Expert advisory board for German migration foundations.
\textsuperscript{3} Other studies paint a less detrimental picture of migration (e.g. DIW Berlin (2008)), but in Germany the threat of demographic change has sparked a public debate about the migration of highly qualified workers (Handelsblatt (2010)).
\textsuperscript{4} Of course, there are also proponents of the beneficial effects of tax competition. Teather (2005), for instance, mentions fiscal restraint, efficiency gains in both businesses and governments and lower tax rates cum tax base broadening.
Luxembourg with a tax wedge\(^5\) of 41.4%, Norway with a wedge of 43% and Sweden with 50.9% (OECD (2010)) for high-earning singles, for example, are able to sustain tax rates above the OECD average (41.1%) and still attract many migrants from other European countries. Immigration to Norway, which is mainly labour immigration from other European countries, has exceeded emigration by more than 40,000 persons in 2009, illustrating this country’s attractiveness despite its high level of redistribution (Statistics Norway (2009)). Of course, the aforementioned tax wedges are still well below those of Germany (53%) and France (53.1%) but the lower tax rates are not accompanied by a similar decrease in publicly mandated social expenditure. Also, indirect taxes such as VAT in Germany and France are way below those of Norway, Sweden and Luxembourg (OECD (2007))\(^6\). This points to high taxation and generous welfare systems in the Scandinavian countries and Luxembourg (see Table 1).

This paper presents a model which shows that under the assumption of altruistic preferences a severe Race to the Bottom is not bound to occur. The model can also offer an explanation why countries with only a small number of poor welfare recipients and a high tax rate such as Luxembourg and Norway can be successful in the competition for taxpayers. The first end is achieved by showing that under a set of plausible assumptions about the altruism of taxpayers tax competition between two countries will lead to a unique Nash equilibrium in taxation that, given sufficient income and sufficiently altruistic sentiments, is non-zero. The second aim is tackled by means of simulations, which show that countries with a low number of welfare recipients in relation to their competitors will attract more taxpayers and will also be able to raise higher taxes. This is a surprising outcome as common theory usually predicts lower tax rates in small countries (see, for instance, Wilson (1991)). I achieve these results in a setting that would lead to zero tax rates in equilibrium in the absence of altruism.

According to various strands of theory, welfare transfers might be desirable for taxpayers for reasons other than altruism, but these concepts show severe weaknesses. First, welfare transfers act as an insurance against the sudden loss of one’s earning ability through, say, unemployment, or, from an ex ante perspective from behind the “veil of ignorance”, as an insurance against the possibility of drawing a bad lot in the lottery of life (Rawls (1971)).

\(^5\) The OECD defines the tax wedge as the ratio between income tax, employer and employee social contributions minus cash transfers to labour costs. The tax wedge does not include additional compulsory contributions to privately managed pension funds or insurances.

\(^6\) Of course, especially Luxembourg and Norway additionally have very high GDP p.c., but in the first case this is mainly due to revenues from international banking and in the latter case to oil production and hence does not accrue to most migrants (OECD (2009)).
Related to the “veil of ignorance” is the idea of parents insuring their children, about whose earning ability they cannot be sure, by establishing and maintaining a welfare transfer system. But these conceptions of uncertainty stand on shaky ground as the risk of becoming unemployed is far lower for well-educated citizens, social downward mobility is quite restricted and children have a high chance of ending up in the same social strata as their parents. Hence, wealthy taxpayers should rather prefer to not insure at all or to insure themselves privately to keep out the “bad risks”. Second, migration costs have been put forward as a reason for the possibility of taxation: Citizens can be taxed simply because leaving the jurisdiction is even more costly for them than paying their taxes. With fast and cheap means of transport and advanced communication via the internet, these costs should be negligible, at least for the group that is most likely to migrate, which consists of the young and well educated professionals. Third, political scientists have argued that internal and external political constraints keep welfare benefits and taxes from being changed at will. This, however, requires ultimately irrational behavior on the part of the electorate and the adherence to non-enforceable agreements between politicians.

The first economic models to deal with the tax erosion issue were mainly focused on tax competition within federations. The analysis was focused on externalities from taxation and migration, possible tax and transfer mechanisms to account for these externalities and general equilibrium effects from tax competition. In this context, country preferences and migration costs were also discussed, with the latter being widely accepted as being present in the labour market. Zero taxation does only occur in the absence of a language barrier, cultural differences or any costs of migration at all. Altruism was also considered, but it did not prove sufficient to generate efficient outcomes of tax competition.

European integration has sparked research on the reaction of taxes and welfare states to increasing mobility and free labour market access. Weichenrieder and Busch (2007) present a framework in which zero taxation is the outcome in a world with perfectly mobile, non-altruistic taxpayers and immobile poor who decide on the level of taxation. The same result occurs under the assumption of immobile, altruistic taxpayers who can also decide on the tax

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7 In 2009, only 5.3% of the German unemployed had an academic education (Bundesagentur für Arbeit (2009)).
8 See, for instance, Jonsson et al. (2009). They find a high degree of social reproduction (children ending up in the same social class as their parents) in four industrialized countries, although they attribute this effect to occupational reproduction rather than to class reproduction.
9 E.g., Thompson (2009) finds a high willingness to migrate in this group.
10 Amongst others, this has been argued by Basinger and Hallerberg (2004) and Gilardi and Wasserfallen (2010).
12 E.g., Sinn (2003).
rates, and perfectly mobile poor. They predict a full Race to the Bottom, so in equilibrium, no taxation is possible in both cases. In this paper, I will challenge the result of Weichenrieder and Busch (2007) by sticking with the assumption of perfectly mobile taxpayers who cannot decide on tax levels, but by also making these taxpayers altruistic. Furthermore, I will explicitly model the competition between tax revenue maximizing jurisdictions and the migratory response of taxpayers based on several forms of altruism, something that has not been done in the literature before.

As pointed out earlier, empirical research on the issue of welfare-state and tax induced migration has produced mixed results. While some researchers claim to have found a relationship between taxes, welfare state generosity and migration both in the US and the European Union, others refute this claim on empirical grounds and attribute perceived tax and migratory effects to inflation or other factors\(^\text{13}\). More qualitative evidence comes from a report by the German prognos research group published in 2008 which finds that taxation is a driving factor for about 47% of the business and 17% of the academic personnel who consider migrating from Germany. The German Institute for Economic Research (DIW Berlin), however, released a report in the same year stating that most German emigrants will eventually come back from abroad and are then better qualified, earn higher wages and subsequently pay more taxes.

Obviously, there is no empirical consensus on the issues of the Race to the Bottom of welfare states and taxpayer migration. The model I will propose in this paper sets out to explain why the absence of a definitely noticeable tax-induced migration and the continued existence of welfare states are in line with the preferences of perfectly mobile taxpayers.

2. **Concepts of Altruism**

The idea that humans are not guided purely by economic considerations, but by a divinely commanded, innate or acquired concern for their fellow man has been around for millennia\(^\text{14}\). However, incorporating this notion into an economic model requires modifications to the

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\(^{14}\) Charity is demanded in both the Old and New Testament and the Qu’ran. The causes and effects of charity are an important issue in Thomas Aquinas’ “Summa Theologiae” (Ney (2006)), and Immanuel Kant (1785) deduces the duty of charity from his notion of the categorical imperative.
concept of the self-interested, coldly calculating homo oeconomicus. This section presents three different approaches to the idea of altruism.

A first concept is pure altruism. One could imagine that taxpayers receive utility from the welfare benefit, financed by their taxes, an individual poor person in their jurisdiction receives. This could be because they are genuinely “good” people who feel empathy for the lot of others. Or they could just as well be motivated by purely egoistic reasons: Having beggars off the streets, reducing poverty-related crime or, generally speaking, “keeping the masses quiet” are motivations that can also be captured by a preference for income redistribution. Furthermore, it could be the case that the productivity and/or income of a rich taxpayer depends on the human capital of the poor (he might need educated workers for his factories, as assistants, etc.), so income redistribution to pay for the poors’ education is rational for taxpayers even though they do not care about the living conditions of the poor per se.

But pure altruism does not depend on one’s own contribution in large societies and therefore suffers from free-riding issues related to public goods. Bernheim (1986) argues that this kind of altruism would produce results which are incongruent with the results observed in real life. Furthermore, this kind of altruism is very sensitive to the valuation of the public good. The first effect of pure altruism has already been noted, albeit not in its negative implications, by the German moral philosopher J.G. Fichte in the late 18th and 19th century, who stated that a good person wants good acts to take place and does not care by whom they are performed (no year). Experimental evidence also points out this shortcoming of a purely altruistic motivation.

A second concept is inequity aversion, which has come up in recent research as a possible driving force behind income redistribution. Here, it is not concern for the income of others, but worries about the difference between one’s own income and that of others which leads to voluntary giving. Fehr and Schmidt (1999), as well as Bolton and Ockenfels (2000) find that equity preference can explain a wide range of experimental outcomes, ranging from completely selfish behavior to full cooperation, in public good and dictator games. In addition

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15 In Germany, wealthy taxpayers like Dietmar Hopp (one of the founders of SAP) and a club of 50 millionaires have demanded higher income taxation instead of social welfare cuts to overcome the current dire fiscal situation (Abendblatt (2010) and ZDFinfoKanal (2010)).
16 This idea is also found in Alesina and Giuliano (2009) and earlier work.
18 E.g. Palfrey and Prisbrey (1997) fail to find purely altruistic preferences in a voluntary contributions experiment.
to fairness considerations as expressed in Bolton and Ockenfels’ (2000) idea of “equity and reciprocity”, the same basically egoistic motivations for an aversion to inequity as in the pure altruism case apply.

Finally, a third approach to motivate altruistic behavior is *warm glow altruism* which has been put forward by Andreoni (1990) and tested, amongst others, by Palfrey and Prisbrey (1997). According to the warm glow theory, the act of giving generates utility for an individual as he or she is receiving a good feeling from being generous. Neither the utility of the donation recipient nor the total provision of a good financed by donations matters, only the individual contribution. The opposite effect, a “cold chill”, analytically works in the same way with an inverted sign and captures the pangs of conscience generated by not adhering to a social norm, to the direct request of a fund raiser or similar external demands for charity.

One could argue that warm glow can only be applied to truly voluntary giving, like donations to charities, but there are reasons why it might also be reasonable to consider warm glow preferences in connection with taxation. A mobile individual does not have to take the tax rate of a country as given but can migrate to another country, and hereby choose his or her own preferred contribution to the welfare state. Therefore, in a setting with competing countries, taxes become, at least to some extent, a choice variable for individuals. Civic duty, resulting from an upbringing in an environment in which taxation and redistribution is the acknowledged social norm, might also induce individuals to feel a warm glow (or at least avoid a cold chill) by paying their taxes as they are fulfilling their perceived obligations towards society. Finally, as Schlicht (1998) argues in his self-attribution theory, the mere fact of doing something might lead one to like it even if another salient reason, such as compulsion, is at hand. Hence, taxpayers might either try to fulfill an obligation by paying taxes or satisfy an acquired “taste” for redistribution.

Having established the applicability of theories of altruism on taxation, I will now turn to a model incorporating altruistic preferences in a tax competition framework.

### 3. A Model of Tax Competition with Altruistic Taxpayers

In this section, I will present a model of two states competing for mobile altruistic taxpayers which produces unique, non-zero pure strategy Nash equilibria for tax levels. A Race to the

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19 Particularly in Germany, these effects could both be at work as minimizing the personal tax load through the myriads of deductions laid down in the German tax code can be considered the social norm, so paying the “normal” amount already amounts to something akin to voluntary giving.
Bottom is no longer inevitable, although, depending on the specific form of the altruistic preferences, the number of welfare recipients has a negative effect on the possibility of taxation.

There are two countries which try to maximize the welfare transfer they can offer to their local, immobile poor by maximizing tax revenue. This transfer constitutes the only income for this group. A tax is collected from a continuum with mass 1 of costlessly mobile taxpayers with an exogenously given income who incorporate some form of altruism, to be specified later, in their utility functions. These taxpayers are heterogeneous w.r.t. to their preference for redistribution of income. Given the strength of his or her altruistic feeling, the tax rate and the number of immobile poor in each country, each taxpayer decides where to settle. In the case of identical countries and tax levels, one half of the total number of taxpayers will end up in each country. This implies that taxpayers are unable to coordinate their migration decisions even if it were beneficial for them to all settle in the same country.

The model is therefore solvable by backward induction: Each country non-cooperatively chooses a tax level anticipating the taxpayers’ reaction, and then taxpayers choose their preferred country.

The optimization problem for country $i$ is thus

$$\max_{b_i} b_i * m_i(x, b_i, b_{j \neq i}, n_i, n_{j \neq i})$$

where, for $i,j=[1,2]$, $x$ is the exogenously given income of each taxpayer with $x > 0$, $b_i$ is the tax rate in country $i$ with $0 \leq b_i \leq x$, $m_i$ is the number of taxpayers who settle in country $i$, and $n_i$ is the number of poor in country $i$. The utility of a taxpayer can be formalized as

$$U = V(\text{disposable income}) + \alpha * W(\text{altruism})$$

$V$ is a strictly concave function, while $W$ only has to be concave. For simplicity, from now on a quasi-linear utility function is assumed, i.e. $V$ is assumed to be the square root and $W$ to be simply linear. The tendency (but of course not the magnitude) of the results is the same for all strictly concave functions of $V$ and $W$, though.

$\alpha$ is the strength of the altruistic motivation, distributed between 0 and 1 (not including the borders)$^{20}$, so no taxpayer is either completely devoid of altruism or values his altruism.

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$^{20}$ Experimental evidence (e.g. Andreoni and Miller (2002) and Fehr and Schmidt (1999)) supports the idea of heterogeneous individuals w.r.t. altruistic preferences.
function as much as his disposable income function. This assumption is needed to ensure the stability of the Nash equilibrium with pure altruism and inequity aversion preferences. In the following sections, it is assumed that $\alpha$ has a truncated normal distribution between 0 and 1 with mean $\mu$, $0 < \mu < 1$, and standard deviation $\sigma > 0$. Note that taxpayers do not have a home country bias, i.e. they do not prefer one country over another ex ante, and that they are costlessly mobile, which leads to a zero taxation outcome in the absence of altruism. Taxpayers also only care about welfare transfers in the country they settle in. This assumption will be relaxed later.

The number of taxpayers settling in each country can be determined by calculating a cutoff level of $\alpha, \alpha^*$, which corresponds to the taxpayer’s $\alpha$ who is just indifferent between both countries:

$$\alpha^* = \frac{V(\text{disposable income in } j) - V(\text{disposable income in } i)}{W(\text{altruism in } i) - W(\text{altruism in } j)}, i \neq j$$

Assuming the same number of poor in each country, taxpayers with $\alpha > \alpha^*$ will settle in the country with higher taxes and welfare transfers, while those with $\alpha < \alpha^*$ choose the one with lower taxation and transfer levels.

Several formulations of the altruistic preferences can now be imagined, based on the different concepts of altruism presented in the previous section. These are summarized in Table 2. As can be seen from these formulas, in the warm glow case utility from transfers only depends on the own contribution. In the pure altruism case, the size of the transfer to each individual poor person matters, while in the inequity aversion case, the sign of $W$ changes from + to – (as income differences create disutility) and disutility depends on the difference between disposable income and the transfer to each individual welfare recipient.

Given this setup, unique pure strategy Nash equilibria values for taxation can be found for each case which differ from the Race to the Bottom result of zero taxation. In each case, mixed strategy Nash equilibria also exist, but the focus in on pure strategy Nash equilibria as the notion of countries randomizing their tax rates over a certain range is not universally accepted. However, in theory it would also suffice just to show that zero taxation is not a stable pure strategy Nash equilibrium to rule out the outcome of zero taxation in a world with altruism.

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21 Including the zero bound makes it possible for a country to always attract marginally more taxpayers.
3.1 No Altruism

To illustrate the typical results of tax competition models lacking mobility constraints or other competition-reducing assumptions, I will shortly cover the results of the model in the absence of altruism. In this case, the utility of a taxpayer will only depend on his or her disposable income in a given country, that is, $U_i = \sqrt{x - b_i}$. Obviously, all taxpayers will choose to move to the country that levies the lowest tax rate as their preferred tax rate is zero.

The implication for the tax competition between two states is that a country can attract all taxpayers by marginally undercutting the other’s tax rate. This will lead to ever smaller tax rates as countries continuously undercut each other, resulting in the only stable outcome of zero taxation in both countries. Any country setting a positive tax rate will not have a tax base to charge taxes from. This is also the result in the Weichenrieder and Busch (2007) model in the case of perfectly mobile, non-altruistic taxpayers and in line with the standard non-collusive result of Bertrand competition.

3.2 Warm Glow

In the warm glow setting, a taxpayer receives utility from his own tax payment, regardless of the resulting individual transfer to each welfare recipient. Taxpayer utility in country $i$ is now given by

$$U_i = \sqrt{x - b_i} + \alpha * b_i$$  \hspace{1cm} (1)

and $\alpha^*$ is determined by

$$\alpha^* = \frac{\sqrt{x - b_j} - \sqrt{x - b_i}}{b_i - b_j}, i \neq j$$  \hspace{1cm} (2)

Due to symmetry between the two countries, both countries will attract exactly one half of the taxpayers in a symmetric pure strategy Nash equilibrium, and will set the same tax rate. Tax revenue $\pi_i$ in country $i$ is given by

$$\pi_i = b_i * m_i = b_i * \frac{\Phi \left( \frac{\alpha^* - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)}{\Phi \left( \frac{1 - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)}$$  \hspace{1cm} (3)

where the last term of the equation, representing the cumulative distribution function (cdf) of a truncated normal distribution, determines the number of taxpayers in country 1 according to the distribution of $\alpha$ ($\Phi$ is the cdf of a standard normal distribution). The optimal tax rate,
as will be proven later, corresponds to the preferred tax rate of the taxpayer with mean altruistic preferences (henceforth referred to as “mean taxpayer”), i.e. with \( \alpha_m = \mu^{22} \). Note that as a truncated normal distribution is being used, \( \mu \) has to be transformed to \( \mu^* \):

\[
\mu^* = \Phi^{-1}\left(\frac{1}{2} \left( \Phi\left(\frac{1 - \mu}{\sigma}\right) - \Phi\left(\frac{0 - \mu}{\sigma}\right) \right) + \Phi\left(\frac{0 - \mu}{\sigma}\right) - \mu \right)
\]

(4)

From (1), by substituting \( \alpha_m \) for \( \alpha \), taking the first derivative w.r.t. \( b_i \) and solving for \( b_i \) the optimal tax rate \( b^* \) is given by

\[
b^* = x - \frac{1}{4\alpha_m^2} = x - \frac{1}{4(\mu^*)^2}
\]

(5)

For a positive \( b^* \) to exist,

\[
\mu^* > \frac{1}{2\sqrt{x}}
\]

(6) shows that if \( \mu^* \) is sufficiently high in relation to \( x \), \( b^* \) will be positive. As \( \mu^* \) is a function of \( \mu \), this corresponds to a higher mean of altruism in the population of taxpayers.

The Nash equilibrium tax rate as given by (5) displays several interesting and logical characteristics: First, a full Race to the Bottom is obviously avoided as both countries will choose the same positive tax rate if (6) is satisfied, which becomes more probable as \( \mu \) and \( x \) increase. Second, an increase in the mean value of the altruistic preferences distribution will increase the optimal tax rate as can be seen in (5), while the effect of an increase in \( \sigma \) depends on whether \( \mu \) is above (negative effect) or below (positive effect) 0.5. The reason for this mechanism is that if \( \alpha \) is uniformly distributed, the optimal tax rate will be the one preferred by the taxpayer with \( \alpha_m = 0.5 \). As the standard deviation of the normal distribution increases, it becomes closer in form to the uniform distribution, and therefore the optimal tax rates converge. These two effects can be seen in Figure 2, where it is noticeable that even very low values of the mean and/or the standard deviation support positive tax rates. Third, due to the quasi-linear formulation of preferences, with a sufficiently large \( x \) all further increases in income will be taxed away to provide welfare transfers.

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22 More generally, \( b_i \) will be set such that \( \frac{\partial y}{\partial b_i} = \alpha_m \ast \frac{\partial y}{\partial b_i} \), i.e. the marginal utilities from disposable income and welfare benefits must be equal for the mean taxpayer.
Proposition 1: Assuming warm glow preferences and given a sufficiently large $\mu^*$ and $x$, a unique pure strategy NE for taxation exists in which both countries choose the non-zero tax rate that corresponds to the one preferred by the mean taxpayer.

A formal proof can be found in the Appendix. This result is not affected in any way by the number of welfare recipients as taxpayers value only the size of their own contribution, not the impact it has on the individual welfare payment. Note that as long as there are some altruistic taxpayers and $x > 0.25$, zero taxation is still not a possible outcome of tax competition even if 6) does not hold as at least taxpayers with an $\alpha$ of 1 will still be in favour of some taxation $^{23}$. A country will be able to generate tax revenues by setting a non-zero tax rate and attracting the most altruistic taxpayers. However, this will only be a NE in mixed strategies as a country slightly undercutting the other’s tax rate will be able to attract all taxpayers who would prefer a zero-level of taxation.

3.3 Pure Altruism

In the pure altruism setting, the welfare transfer to each individual recipient matters for taxpayers. Hence, taxpayer utility in country $i$ is now given by

$$U_i = \sqrt{x - b_i} + \alpha \cdot \frac{b_i m_i}{n_i}$$

(7)

Here, it will be assumed that $n_1 = n_2 = n$, that is, the countries are symmetric with respect to the number of their poor inhabitants. The indifferent taxpayer now has an $\alpha^*$ of

$$\alpha^* = \frac{\sqrt{x - b_j} - \sqrt{x - b_i}}{\frac{b_i m_i}{n} - \frac{b_j (1 - m_j)}{n}}, \quad i \neq j$$

(8)

A country setting its tax rate must now be aware that rational taxpayers all want to settle in the same country, as this would maximize the transfer to the poor and hence their utility from welfare benefits. However, because they cannot coordinate, they can only estimate where most taxpayers will end up, and then all move to that country. If both countries set the same tax rate, they evenly distribute themselves over the two jurisdictions as they have no indication where more than half of them will settle.

$^{23}$ The marginal utility from welfare benefits is 1 for an individual with $\alpha = 1$, so he or she prefers taxation if the marginal utility from disposable income is smaller than 1, which in the quasi-linear setting is the case if $x > 0.25$. 
Thus, in an equilibrium, both countries will set the tax rate that is preferred by the taxpayer with an $\alpha$ of $\mu^*$, that is, with exactly mean altruistic preferences. The reason is that any tax rate except the equilibrium one is preferred by less than half of the taxpayers, so a migration will take place which will make the country receiving the migrants more attractive: Even though some taxpayers migrate from the country with an unchanged tax rate, more will leave the one with altered taxes. This can be seen from the first derivative w.r.t. $b_i$ of (8) for $m_1 = m_2 = 0.5$:

$$\frac{\partial \alpha^*}{\partial b_i} = \frac{1}{2\sqrt{x - b_i}} + \frac{1}{2n} \left( \frac{\sqrt{x - b_i} - \sqrt{x - b_j}}{2n} \right)^2, \quad i \neq j$$

(9)

This function is always positive if $b_i > b_j$ and negative if $b_i < b_j$. Thus, as $\alpha^* = a_m$ in equilibrium, any change in the tax rate will lead to a loss of taxpayers in country $i$ as $i$ attracts exactly 1 minus an increasing function of $\alpha^*$ taxpayers in the first and an increasing function of $\alpha^*$ taxpayers in the latter case.

The higher number of taxpayers makes the country maintaining the equilibrium tax more attractive, which in turn leads to even more migration from the other one, so in the end all taxpayers crowd into the country which has maintained the equilibrium tax rate. This is evident from the derivative of $\alpha^*$ w.r.t. $m_i$

$$\frac{\partial \alpha^*}{\partial m_i} = \frac{(b_i + b_j)}{n} \left( \frac{\sqrt{x - b_i} - \sqrt{x - b_j}}{n} \right)^2, \quad i \neq j$$

(10)

which, as in the last paragraph, alters the indifferent taxpayer unfavourably for country $i$. In equilibrium, the only thing a country can achieve by altering its tax rate is to lose all its taxpayers to the other country. The equilibrium tax rate is therefore given by substituting $a_m$ for $\alpha$ in (7), setting $m_i$ to 0.5 and maximizing the equation w.r.t. to $b_i$. This yields an optimal tax rate of

$$b^* = x - \frac{n^2}{(\mu^*)^2}$$

(11)

The important finding here is that, although there is still a positive tax rate if the mean taxpayer is sufficiently altruistic, the possibility of taxation is now negatively affected by the
number of poor. \( \mu^* \) has to be sufficiently high in relation to \( x \) and \( n \) to ensure positive taxation, i.e. positive taxation occurs if

\[
\mu^* > \frac{n}{\sqrt{x}} \tag{12}
\]

Proposition 2: Assuming pure altruism preferences and given a sufficiently large \( \mu^* \) and \( x \), a unique pure strategy NE for taxation exists in which both countries choose the non-zero tax rate that coincides with the one preferred by the mean taxpayer. The equilibrium tax rate is decreasing in the number of poor.

The proof can be found in the Appendix. This equilibrium, however, is unstable. Slight deviations from the equilibrium \( b \) and \( m \) will lead to an infinite cycle of tax alterations which can only by chance lead to a return to the equilibrium. Still, zero taxation is not a stable equilibrium of this model, so pure altruism will still prevent a complete Race to the Bottom if \( (12) \) holds. Also, even if \( (12) \) does not hold, there are still Nash equilibria in mixed strategies if \( x > \frac{4n^2}{m^2} \), as in this case at least the most altruistic taxpayers still prefer some welfare transfers. \( \bar{m} \) is the highest attainable number of taxpayers by a country raising its tax rate above zero. Thus, if this parameter is very small due to the distribution of \( a \), even the most altruistic taxpayers do not benefit from taxation and welfare transfers.

### 3.4 Inequity Aversion

Under the assumption of altruism motivated by inequity aversion, taxpayers care about the difference between their disposable income and the income (consisting solely of the transfer) of each individual welfare recipient. Due to the migratory responses pointed out in the previous section, a country in the case of inequity aversion also has to cater for the preferences of the mean taxpayer to avoid losing all its tax base to the other country.

A taxpayer’s utility in country \( i \) is represented by

\[
U_i = \sqrt{x - b_i} - \alpha \left( x - \sqrt{x - b_i} - \frac{b_i m_i}{n_i} \right) \tag{13}
\]

and the indifferent taxpayers has an \( \alpha \) of

\[
\alpha^* = \frac{\sqrt{x - b_i} - \sqrt{x - b_j}}{\left( b_i + \frac{b_i m_i}{n_i} \right) - \left( b_j + \frac{b_j (1 - m_j)}{n} \right)}, i \neq j \tag{14}
\]

13
Thus, using the same method as in the previous two sections, the preferred tax rate of the mean taxpayer if half the taxpayers settle in one country is

\[ b^* = x - \frac{n^2}{(\mu^* + 2\mu^*n)^2} \]  

(15)

Deviations are non-optimal as the implications of a change in the tax rate are the same as for (9) (deviating from \( b^* \) always reduces the number of taxpayers for a country):

\[ \frac{\partial a^*}{\partial b_i} = \frac{1}{2\sqrt{x - b_i}} + \frac{1 + \frac{1}{2n}}{(b_i + \frac{b_i}{2n}) - (b_j + \frac{b_j}{2n})} \left( \frac{1}{(b_i + \frac{b_i}{2n})} - \frac{1}{(b_j + \frac{b_j}{2n})} \right)^2, i \neq j \]  

(16)

This, similar to (10), leads to further migratory responses, which in the end drive out all taxpayers from the deviating country, because of

\[ \frac{\partial a^*}{\partial m_i} = \frac{(b_i + \frac{b_i}{n})(\sqrt{x - b_i} - \sqrt{x - b_j})}{\left( \frac{b_i}{n} + \frac{b_i}{n} \right) - \left( \frac{b_j}{n} + \frac{b_j(1 - m_i)}{n} \right)} \]  

(17)

Again, \( b^* \) is decreasing in \( n \), but only up to a certain amount, and slower than in the pure altruism case. This is graphically shown in Figure for varying \( \mu \) and \( n \). As \( n \) approaches infinity, there is still a chance of taxation with a sufficiently high \( \mu \). More specifically, for positive taxation to occur,

\[ \mu^* > \frac{n}{2 \sqrt{x} + 1} \]  

(18)

**Proposition 3:** With inequity aversion in the taxpayers’ utility function, a unique pure strategy NE for taxation exists in which both countries choose the non-zero tax rate that is preferred by the mean taxpayer if \( \mu^* \) and \( x \) are sufficiently large. The equilibrium tax rate is decreasing in the number of poor, but not as strongly as in the pure altruism case.

The proof, which can be found in the Appendix, is similar to the pure altruism case. The inequity aversion is unstable for the same reason as the pure altruism one, but once again
mixed strategy Nash equilibria exist and zero taxation is not an equilibrium in this setting unless \( x < \frac{4}{(14m)^3} \).

### 3.5 Spatial Altruistic Preferences

As shown in the previous sections, altruistic preferences can lead to non-zero taxation even in the absence of migration costs and non-cooperation between countries. But as Pauly (1973) has proposed, altruistic feelings do not have to stop at borders. Especially in the case of a federation, it is reasonable to assume that citizens in one state also care about the well-being of citizens in other states, at least to some extent\(^{24}\). However, welfare benefits in one state create a positive externality for other states in the sense that taxpayers in state 1 benefit from transfers in state 2 without having to pay for them. In the model presented in this paper, in contrast to Pauly’s (1973), taxpayer mobility in connection with spatial altruistic preferences only affects the possible scope of taxation if altruism is not diminished by distance at all. The utility of a taxpayer in country \( i \) is now given by

\[
U_i = \sqrt{\text{disposable income in } i + \alpha \ast (altruism in } i) + \beta \ast \alpha \ast (altruism in } j)
\]

\( \beta \) is a weighting factor for spatial preferences: If \( \beta = 1 \), individuals care equally about the welfare benefits in both countries, if \( \beta = 0 \), they only consider the welfare benefits in the country they settle in as a ‘good’.

Note that in the case of warm glow altruism, spatial preferences do not play a role as a taxpayer only values his or her own contribution, which excludes the possibility of caring about welfare transfers in the other country as these are not financed by him- or herself.

In the other two cases, incorporating spatial preferences will not affect the optimal tax rates unless \( \beta = 1 \), because in this case, taxpayers always prefer to relocate to the country with lower taxes as they reap the whole benefit from higher transfers in the high tax country without having to pay for them. This will result in a full Race to the Bottom as described in section 3.1. For a smaller \( \beta \), the same mechanism that prevents deviations from the NE will take place as in the setting without spatial preferences: \( \beta \) does not influence the preferred tax

\(^{24}\)This is mirrored by a central government’s concern for each individual state. E.g., federal redistribution is taking place in Germany through the “Länderfinanzausgleich” and also in the European Union on a larger scale.
rate of the mean taxpayer in his host country, and the effects leading to the stable equilibrium are not altered. E.g., (8) becomes

\[
a^* = \frac{\sqrt{x - b_j} - \sqrt{x - b_i}}{b_j (1 - m_i) + \beta \left( \frac{b_j (1 - m_i)}{n} - \frac{b_i m_i}{n} \right)} , \quad i \neq j
\]

(19)

which, as well as its derivatives, differs only in magnitude, not in sign.

### 3.6 Asymmetric Countries

A further interesting case is the outcome of tax competition with asymmetric countries w.r.t. the number of welfare recipients. One could expect that countries which have to support a smaller number of poor people have an advantage in the competition for taxpayers, at least if the income per welfare recipient matters as in the inequity aversion and pure altruism cases. Taxation under the assumption of a warm glow from paying taxes is not affected, as a taxpayer’s utility depends neither on the number of other taxpayers in his jurisdiction, nor on the income of the poor. Since solving the model involves polynomial equations of high degree if the number of poor differs between countries, I will present simulation results in this section.

Starting from an initially given distribution of taxpayers, these results were achieved by letting the countries alternately choose their optimal (=tax revenue maximizing) tax rate while taking into account the migratory responses. Using this mechanism, a Nash equilibrium occurs if no country has an incentive to deviate from its tax rate and the distribution of taxpayers between countries remains stable. The results for varying \( \mu \) are shown in Figure 4, the case of varying \( n \) is shown in Figure 5.

Again, tax rates generally increase in \( \mu \), and inequity aversion, the stronger form of altruism, produces higher equilibrium outcomes. Increasing \( n_2 \) makes the differences more pronounced, as the tax rates under pure altruism react more strongly than under inequity aversion. Furthermore, the advantage of country 1 increases in \( n_2 \) which widens the gap between the two countries’ tax rates.

It is also important to note (not shown in the Figure 4) that most taxpayer locate in the high tax/low poverty country, so having only few welfare recipients in comparison to other
countries gives a twofold advantage: On the one hand, higher tax rates are sustainable in equilibrium, and on the other hand, most tax payers prefer the high tax country.

Proposition 4: If countries differ w.r.t. the number of welfare recipients, a unique NE exists with purely altruistic and inequity aversion preferences in which the country with the lower number of poor will set higher taxes and attract more taxpayers.

In this model, a smaller number of poor is also an indicator for a smaller country size. It has been argued, for instance by Chatelais and Peyrat (2008), that small countries are drivers of tax competition as their benefits (attracting taxpayers) from lowering the tax rate outweigh the drawbacks (lower tax rates) in relation to their small GDP, something that does not hold for big countries. This stands in contrast to the result presented here: Small countries are able to sustain higher taxes than large ones, so they should not play the role of the driving force behind tax competition by undercutting large countries’ tax rates.

4. Discussion and Conclusion

Existing models of tax competition predict that in the absence of migration costs and other barriers to migration, countries will be forced to lower taxes and dismantle their welfare states. In contrast, the model presented in this paper can explain the “stickiness” of taxpayers and the absence of a full Race to the Bottom due to migratory pressures by assuming an intrinsic altruistic motivation of taxpayers. As shown, the reason behind including a seemingly altruistic term in an individual’s utility function does not have to be altruistic at all, but this does not affect the results. For various formulations of the utility function, a destructive tax competition is not bound to occur. Without further assumptions, there is nothing to be gathered about the efficiency of the chosen tax policies from this model. Assuming utilitarian countries, the resulting tax rates definitely fall short of the associated full income equalization. The model implicitly assumes a linear utility function of welfare recipients w.r.t. income, but modeling the poor’s utility function in a different way will not affect the results unless countries suddenly start taking into account both the taxpayers’ and welfare recipients’ utility.

The result put forward in Proposition 1 is the most basic case, with a utility function that has been used to include altruism in other models, even though, as I pointed out, the formulation is more in line with the warm glow idea. The existence of a positive welfare transfer depends only on the income and the distribution of altruistic preferences. If taxpayers are suitably
characterized by this utility function, it is fair to say that the high incomes in Western countries and the degree of social cohesion and identification with the political system, which could be used as a proxy for the strength of altruistic preferences, are sufficient to maintain welfare states. Furthermore, having the mean taxpayer’s preferred policy as the outcome of the tax competition can be seen as being in line with the Median Voter theorem from the Political Economy literature. With the truncated normal distribution used in this paper, mean and median coincide, but as the driving factor behind the equilibria is the fact that diverging tax rates will attract less than half of the tax base, one can imagine that skewed distributions will lead to the countries implicitly choosing the median taxpayer’s preferred policy. Therefore, in the light of the median voter theorem, the outcome of international tax competition should also be politically feasible. In addition, a dispersion of preferences is increasing tax rates in the likely case that a population is on average less altruistic than a uniform distribution would suggest. This heterogeneity of preferences is experimentally found by Andreoni and Miller (2002), who also state that three quarters of their test subjects display some form altruistic behavior.

In Propositions 2 and 3, one sees that the possibility of redistribution depends to a varying extent on the number of poor in the first place. Now, not only income and preference distribution affect equilibrium taxes, but also the number of welfare recipients. For Western countries, one can assume a high ratio of net taxpayers to welfare recipients, so redistribution should also be possible under the weaker assumption of purely altruistic preferences. In a tax competition game between poor countries, however, the sheer amount of eligible welfare recipients and the low impact of an additional unit of money on the individual payments should make redistribution all but impossible under Proposition 2. Inequity aversion, which could be linked to fairness considerations and social norms which are less likely to exist in poorer countries, seems to be less applicable in the case of tax competition between developing countries.

Pitting poor and rich countries against each other, Proposition 4 is congruent with observations from real life: Poor countries struggle to keep their taxpaying population and set lower taxes, while rich countries are attractive as they only have to distribute their tax revenues to a smaller number of welfare recipients. E.g., the tax wedge was 60.5% in Belgium.

25 An economic model dealing with the state-directed strengthening of these factors is put forward by Konrad (2007). In his model, countries can invest in the loyalty of their taxpayers which alters the outcome of tax competition between countries.

26 Very limited altruistic feelings, however, would lead to negative taxation and hence redistribution from the bottom to the top in this model.
in 2009 for high-earning singles, but only 20.8% in Mexico and 34.9% in Poland (OECD (2010)). Even between wealthy countries, this effect should be visible, which could explain some of the pull high tax/low poverty countries such as Sweden with a positive net migration of about 60,000 in 2009 (Statistics Sweden (2010)) and Norway exert on European migrants. Proposition 4 also stands in contrast to the common result that small countries drive tax competition by undercutting their neighbours’ tax rates.

Having stated the implications of these propositions, one should be aware that the model is mainly applicable to economically similar countries with politically and culturally similar inhabitants. The distribution of altruistic preferences certainly varies between Western countries, and even more so between EU, Eastern and Asian countries\(^27\). The altruistic mobile tax base should be seen, also due to migration costs which are of course more important for migrants from other cultures and continents, in a European or at least Western context. Furthermore, tax adjustments are long-term processes, so one should not expect to see the predicted results at this time in European policy, but rather adjustments towards equilibrium. As there are high tax countries which are able to attract taxpayers from other high tax and also low tax countries, this seems to be the case.

Of course, migration decisions and the scope of welfare states do not depend on tax differentials alone. As various studies suggest, countries can attract migratory flows through a host of other positive characteristics\(^28\), and the generosity of welfare states is also dependent on political and economic factors. But the reasonable assumption of altruistic preferences when it comes to paying taxes can help to explain why the specter of the Race to the Bottom of welfare states has so far failed to materialize.

\(^{27}\) A recent Eurobarometer poll shows that 53% of all respondents hold national governments responsible for reducing poverty (European Commission (2009)). On a national level, this figure varies greatly, however (24% in France, 85% in Bulgaria).

\(^{28}\) See for instance Van Dalen and Henkens (2007), DIW Berlin (2008) and forthcoming work by Poutvaara.
Bibliography


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**Statistical Resources**


Tables and Figures

<table>
<thead>
<tr>
<th>Country</th>
<th>Tax wedge for a single worker, no kids, earning 167% of the average wage in %</th>
<th>Net publicly mandated social expenditure in % of GDP</th>
<th>Average indirect tax rate in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>60.5</td>
<td>26.2</td>
<td>15.1</td>
</tr>
<tr>
<td>Denmark</td>
<td>48.6</td>
<td>23.9</td>
<td>26.0</td>
</tr>
<tr>
<td>Finland</td>
<td>48.2</td>
<td>22.6</td>
<td>19.9</td>
</tr>
<tr>
<td>France</td>
<td>53.1</td>
<td>29.9</td>
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<tr>
<td>Germany</td>
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<td>Luxembourg</td>
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<tr>
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<td>20.8</td>
<td>9.0</td>
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</tr>
<tr>
<td>Norway</td>
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<tr>
<td>United Kingdom</td>
<td>37.0</td>
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<td>12.8</td>
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<tr>
<td>OECD average</td>
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<td>20.2</td>
<td>15.1</td>
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</table>

Table 1: Tax wedges and welfare expenditures for selected OECD countries. (OECD (2007/2010))

<table>
<thead>
<tr>
<th>Type of altruistic preference</th>
<th>Formal representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Altruism</td>
<td>$W \equiv 0$</td>
</tr>
<tr>
<td>Warm Glow</td>
<td>$W(b_i) = W(b_i)$</td>
</tr>
<tr>
<td>Pure Altruism</td>
<td>$W(b_i, m_i, n_i) = W(b_i \cdot \frac{m_i}{n_i})$</td>
</tr>
<tr>
<td>Inequity Aversion</td>
<td>$W(x, b_i, m_i, n_i) = -W(x - b_i - b_i \cdot \frac{m_i}{n_i})$</td>
</tr>
</tbody>
</table>

Table 2: Formal representation of the different types of altruistic preferences
Figure 1: Emigration, immigration and net migration of “scientists” and “executives” (ISCO 1 and 2) between EU-15 countries, yearly averages between 2005 and 2009. Adapted from Ette and Sauer (2010)

Figure 2: Equilibrium tax levels for warm glow preferences, x=10
Figure 3: Equilibrium tax levels for pure altruism, warm glow and inequity aversion, x=100, σ=0.5
Figure 4: Equilibrium tax levels, $x = 100$, $n_1 = 1$, $n_2 = 1.1$, $\sigma = 0.5$

Figure 5: Equilibrium tax levels, $x = 10$, $n_1 = 1$, $\sigma = 10$
Appendix

Proof of NE for Warm Glow

There exists a unique NE given by

\[ b^* = b_1 = b_2 = x - \frac{1}{4(\mu^*)^2} \]

Assume that country 2 chooses a tax level of \( b^* \). The number of taxpayers in country 1 is given by

\[
m_1 = \begin{cases} 
0.5 \text{ if } b_1 = b_2 \\
\frac{\Phi \left( \frac{\alpha^* - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)}{\Phi \left( \frac{1 - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)} \quad \text{if } b_1 < b_2 \\
1 - \frac{\Phi \left( \frac{\alpha^* - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)}{\Phi \left( \frac{1 - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)} \quad \text{if } b_2 > b_1 
\end{cases}
\]

The number of taxpayers per country has to be 0.5 in equilibrium, therefore the previous 3 terms need to converge towards 0.5 as \( b_1 \) approaches \( b^* \). Hence, by L’Hôpital’s rule,

\[
\lim_{b_1 \to b^*} a^* = \frac{0.5}{\sqrt{x - b^*}}
\]

and thus

\[
\lim_{b_1 \to b_2} \frac{\Phi \left( \frac{\alpha^* - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)}{\Phi \left( \frac{1 - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)} = \frac{\Phi \left( \frac{0.5}{\sqrt{x - b^*} - \mu} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)}{\Phi \left( \frac{1 - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)}
\]

which, substituting for \( b^* \), gives

\[
\frac{\Phi \left( \frac{\mu^* - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)}{\Phi \left( \frac{1 - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)} = 0.5
\]

as the cdf of the mean of a truncated normal distribution is of course 0.5.

In equilibrium, \( \pi_1 \) has to be equal to \( \pi_2 \), and therefore
\[
\lim_{b_1 \to b^*} \pi_1 = \lim_{b_1 \to b^*} \frac{b_1}{\Phi(1 - \mu/\sigma) - \Phi(0 - \mu/\sigma)} = \pi_2 = \frac{b^*}{2}
\]

Obviously, \( b_1 \) converges towards \( b^* \), and it has been shown that the number of taxpayers converges towards 0.5, so what is left to prove is that \( \pi_1 \) is strictly increasing (decreasing) in \( b_1 \) below (above) \( \pi_2 \).

The first derivative of the tax revenue function 3) for country 1 with respect to its own tax level is

\[
b_1 \cdot \frac{\partial \alpha^*}{\partial b_1} \cdot \frac{1}{\sigma} \Phi \left( \frac{\alpha^* - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right) + \frac{\Phi \left( \frac{\alpha^* - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)}{\Phi \left( \frac{1 - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)} > 0 , \text{if } b_1 < b^*
\]

with \( \Phi \) being the probability density function of a standard normal distribution. For \( 0 < \mu < 1 \), the second term always returns positive values. As \( \frac{\partial \alpha^*}{\partial b_1} > 0 \), the whole derivative is positive, which means that an increase in \( b_1 \), if \( b_1 < b^* \), will always increase the tax revenue.

The first derivative of the tax revenue function for country 1 with respect to its own tax level is

\[
-b_1 \cdot \frac{\partial \alpha^*}{\partial b_1} \cdot \frac{1}{\sigma} \Phi \left( \frac{\alpha^* - \mu}{\sigma} \right) + \frac{\Phi \left( \frac{\alpha^* - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)}{\Phi \left( \frac{1 - \mu}{\sigma} \right) - \Phi \left( \frac{0 - \mu}{\sigma} \right)} + 1 < 0 , \text{if } b_1 > b^*
\]

The sum of the first two terms is smaller than -1, so the whole derivative is negative, which means that a decrease in \( b_1 \), if \( b_1 > b^* \), will always increase the tax revenue.

Hence, as \( \pi_1 \) strictly increases in \( b_1 \) if \( b_1 < b^* \) and strictly decreases in \( b_1 \) if \( b_1 > b^* \), and as \( \pi_1 \) converges towards \( 0.5 \cdot b^* \), \( b_1 = b_2 = b^* \) constitutes a unique NE.

**Proof of NE for Pure Altruism**

There exists a unique NE given by

\[
b^* = b_1 = b_2 = x - \frac{n^2}{(\mu^*)^2}
\]

Assume that country 2 chooses a tax level of \( b^* \). Tax revenues in country 1 are
\[ \pi_1 = m_1 * b_1 = \begin{cases} 
0.5 \left( x - \frac{n^2}{(\mu^*)^2} \right) & \text{if } b_1 = b_2 \\
\frac{\Phi(\frac{\alpha^* - \mu}{\sigma}) - \Phi\left(\frac{0 - \mu}{\sigma}\right)}{\Phi\left(\frac{1 - \mu}{\sigma}\right) - \Phi\left(\frac{0 - \mu}{\sigma}\right)} * b_1 & \text{if } b_1 < b_2 \\
1 - \frac{\Phi\left(\frac{\alpha^* - \mu}{\sigma}\right) - \Phi\left(\frac{0 - \mu}{\sigma}\right)}{\Phi\left(\frac{1 - \mu}{\sigma}\right) - \Phi\left(\frac{0 - \mu}{\sigma}\right)} * b_1 & \text{if } b_1 > b_2 
\end{cases} \]

\[ a^* = \frac{\sqrt{x - b_2} - \sqrt{x - b_1}}{\frac{b_1 m_1}{n} - \frac{b_2 (1 - m_1)}{n}} \]

Holding \( m_1 \) constant at 0.5, it is obvious that any tax rate deviating from \( b^* \) will be preferred by less than half of the taxpayers as \( b^* \) is just the tax rate which half of the taxpayers consider to be too low and half to be too high (see (9) and comments). This, in turn, further punishes a deviation from \( b^* \) as

\[ \frac{\partial a^*}{\partial m_1} = \frac{\left(\frac{b_1}{n} + \frac{b_2}{n}\right)\left(\sqrt{x - b_1} - \sqrt{x - b_2}\right)}{\left(\frac{b_1 m_1}{n} - \frac{b_2 (1 - m_1)}{n}\right)^2} \]

Hence, a loss in taxpayers alters the indifferent taxpayer in a strictly unfavourable way for country 1 (see (10) and comments). By deviating from \( b^* \), country 1 changes its tax revenues by

\[ \frac{\partial \pi_1}{\partial b_1} = 0.5 + b_1 \frac{\partial m_1}{\partial b_1} < 0 \]

Thus, \( b^* \) is the optimal tax rate for country 1 given \( b_2 = b^* \).

**Proof of NE for Inequity Aversion**

The proof for a unique pure strategy Nash equilibrium with inequity aversion is similar to the pure altruism case. There exists a unique NE given by

\[ b^* = b_1 = b_2 = x - \frac{n^2}{(\mu^* + 2\mu^* n)^2} \]
Assume that country 2 chooses a tax level of $b^*$. Tax revenues in country 1 are

$$\pi_1 = m_1 * b_1 = \begin{cases} 
0.5 * \left( x - \frac{n^2}{(\mu^* + 2\mu^*n)^2} \right) & \text{if } b_1 = b_2 \\
\Phi\left( \frac{\alpha^* - \mu}{\sigma} \right) - \Phi\left( \frac{0 - \mu}{\sigma} \right) * b_1 & \text{if } b_1 < b_2 \\
\Phi\left( \frac{1 - \mu}{\sigma} \right) - \Phi\left( \frac{0 - \mu}{\sigma} \right) * b_1 & \text{if } b_1 > b_2 \\
\left( 1 - \frac{\Phi\left( \frac{\alpha^* - \mu}{\sigma} \right) - \Phi\left( \frac{0 - \mu}{\sigma} \right)}{\Phi\left( \frac{1 - \mu}{\sigma} \right) - \Phi\left( \frac{0 - \mu}{\sigma} \right)} \right) * b_1 & \text{if } b_1 > b_2
\end{cases}$$

$\alpha^*$, however, is a function of $m_1$:

$$\alpha^* = \frac{\sqrt{x - b_2} - \sqrt{x - b_1}}{(b_1 + \frac{b_1m_1}{n}) - (b_2 + \frac{b_2(1 - m_1)}{n})}$$

Holding $m_1$ constant at 0.5, it is obvious that any tax rate deviating from $b^*$ will be preferred by less than half of the taxpayers as $b^*$ is just the tax rate which half of the taxpayers consider to be too low and half to be too high (see (16) and comments). This, in turn, further punishes a deviation from $b^*$ as

$$\frac{\partial \alpha^*}{\partial m_1} = \frac{\left( \frac{b_1}{n} + \frac{b_2}{n} \right)(\sqrt{x - b_1} - \sqrt{x - b_2})}{\left( (b_1 + \frac{b_1m_1}{n}) - (b_2 + \frac{b_2(1 - m_1)}{n}) \right)^2}$$

Hence, a loss in taxpayers alters the indifferent taxpayer in a strictly unfavourable way for country 1 (see (10) and comments). By deviating from $b^*$, country 1 changes its tax revenues by

$$\frac{\partial \pi_1}{\partial b_1} = 0.5 + b_1 \frac{\partial m_1}{\partial b_1} < 0$$

Thus, $b^*$ is the optimal tax rate for country 1 given $b_2 = b^*$. 