Corrigendum to “Tacit collusion and international commodity taxation”

Andreas Haufler *
University of Munich

Dirk Schindler
University of Konstanz

Guttorm Schjelderup
Norwegian School of Economics and Business Administration

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Haufler and Schjelderup (2004) [henceforth HS] employ a model of dynamic price competition between two firms, each located in a different country $i \in \{1, 2\}$, which decide on whether to collude by agreeing not to export into each other’s home market. Unfortunately there is an error in the original paper, as the prices derived for a defecting firm $i$ in the neighbouring market $j$ are not the profit-maximizing ones in the deviation phase. Despite this error, all results derived in the analysis of HS continue to hold. However, several core equations and the proofs for Propositions 1 and 2 change in substantial ways, and this is corrected in the present note.

The analysis of HS centers around the derivation of critical values of the firms’ common discount factor, above which it is in a firm’s interest to defect from the col-

*Corresponding author. Department of Economics, University of Munich, Akademiestr. 1/II, D-80799 Munich, Germany. Tel. (+49) 89-21803858, e-mail Andreas.Haufler@lrz.uni-muenchen.de
lusive agreement. The critical (relative) discount factor of firm \( i \) is given by 
\[
\tilde{\theta}_i^m = \frac{\pi^E_i}{(\pi^M_i - \pi^D_i)} \quad \forall \ i \in \{1, 2\}, \ m \in \{DP, OP\},
\]
which are compared under the destination principle (DP) and the origin principle (OP). In this expression, \( \pi^E_i \) denotes firm \( i \)'s extra profits from breaking the collusive agreement and exporting to firm \( j \)'s home market in the deviation phase, \( \pi^M_i \) are the monopoly profits of firm \( i \) in its home market \( i \), and \( \pi^D_i \) are firm \( i \)'s total duopoly profits in both markets, which result from the non-cooperative play of firm \( j \) in the punishment phase.

**1. Determinants of cartel stability**

Consider first the destination principle. HS assume that when firm \( i \) deviates from the cartel solution and exports to market \( j \), it will set a price marginally below the price of firm \( j \). This argument overlooks, however, that if firm \( i \) has lower unit costs than its rival (\( c_i < c_j \)), then the profit maximizing producer price of firm \( i \) in market \( j \) is instead the standard monopoly price in this market, denoted 
\[
p_j^i = \frac{1}{2} \left( \frac{a_j}{1+t_j} + c_i \right) < p_j,
\]
which leads to sales 
\[
x_j^i = \left[ a_j - (1+t_j)c_i \right] / 2.
\]
Hence firm \( i \) will underbid firm \( j \) discretely as a result of its cost advantage. In contrast, if firm \( i \) is the high-cost firm, then it remains true that its profit-maximizing price in market \( j \) is just marginally below firm \( j \)'s monopoly price. Hence, two cases must be distinguished in determining the exporting profits of firm \( i \), which coincide when \( c_i = c_j \). Eq. (6) in HS thus changes to
\[
\pi_{E(DP)}^i = \begin{cases} 
[a_j - (1 + t_j)c_i]^2/[4(1 + t_j)] & \text{if } c_i < c_j \\
\alpha_j \left[ \alpha_j + 2(1 + t_j)(c_j - c_i) \right] / [4(1 + t_j)] & \text{if } c_i \geq c_j,
\end{cases}
\]
where \( \alpha_j = a_j - (1 + t_j)c_j \). The correct value of firm \( i \)'s exporting profits in the deviation phase is thus higher than stated in HS, if firm \( i \) is the low-cost firm.

Adopting the convention \( c_i < c_j \), condition (7) in HS must be supplemented to ensure that firm \( i \)'s monopoly price in both markets exceeds the marginal costs of firm \( j \). This implies
\[
\alpha_i - 2(1 + t_i)(c_j - c_i) > 0, \quad \alpha_j - (1 + t_j)(c_j - c_i) > 0.
\]
Duopoly profits in eq. (8) of HS are given correctly, but the earnings of firm \( i \) in markets \( i \) and \( j \) [given in HS two lines above eq. (8)] should read \( (c_j - c_i)[a_i - (1 + t_i)c_j] \) and \( (c_j - c_i)[a_j - (1 + t_j)c_j] \). As a result, the denominator in the expression for the critical
discount rate of the low-cost firm in eq. (9a) needs to be adjusted, along with the change in the numerator [which follows from eq. (6)]. Hence
\[ \hat{\theta}^{DP}_{i} = \frac{(1 + t_{i})[a_{j} - (1 + t_{j})c_{i}]^{2}}{(1 + t_{j})\{\alpha_{j}^{2} - 4(1 + t_{j})(c_{j} - c_{i})[\alpha_{i} + \alpha_{j} - (1 + t_{i})(c_{j} - c_{i})]\}, \quad \text{(9a)} \]
whereas the critical value of the high-cost firm in eq. (9b) remains unchanged.

These changes affect the HS analysis only if the unit costs of the two firms differ (Case C). In this latter case, it is straightforward to derive the corrected equation (11a) from the changes in (9a), and to show that Lemma 1 still holds. This is intuitive as it is the low-cost firm that is more likely to break the collusive agreement, and the incentive to defect is even larger in the corrected analysis due to higher exporting profits \( \pi^{E}_{i} \).

Under the *origin principle*, analogous changes have to be made. The expression for the exporting profits of firm \( i \) will again depend on whether firm \( i \) has lower or higher unit costs than its rival. If firm \( i \) is the low-cost firm, it will charge a monopoly producer price \( p_{j}^{i} = \frac{a_{j}}{1 + t_{i} + c_{i}} \). Hence eq. (12) in HS changes to
\[ \pi^{E(OP)}_{i} = \begin{cases} \frac{[a_{j} - (1 + t_{i})c_{i}]^{2}/[4(1 + t_{i})]}{1 + t_{i}} & \text{if } c_{i} < c_{j} \\ \alpha_{j}\{\alpha_{j} + 2[(1 + t_{j})c_{j} - (1 + t_{i})c_{i}]\}/[4(1 + t_{i})] & \text{if } c_{i} \geq c_{j} \end{cases} \quad \text{(12)} \]

In the duopoly equilibrium, firm \( i \) does not sell \( \alpha_{j}/2 \) units in each market, as is stated on p. 586 of HS [two lines above eq. (13)]. Charging a producer price slightly below \( (1 + t_{j})c_{j}/(1 + t_{i}) \) drives firm \( j \) entirely out of both markets and leaves firm \( i \) with sales of \( a_{i} - (1 + t_{j})c_{j} \) in market \( i \) and \( a_{j} - (1 + t_{j})c_{j} = \alpha_{j} \) in market \( j \). Hence, eq. (13) in HS changes for the low-cost firm \( i \), whose profits in the punishment phase are
\[ \pi^{D(OP)}_{i} = \frac{(1 + t_{j})c_{j} - (1 + t_{i})c_{i}}{1 + t_{i}} \{\alpha_{j} + a_{i} - (1 + t_{j})c_{j}\}. \quad \text{(13)} \]
As a result of the changes in the exporting and duopoly profits of the low-cost firm \( i \), its critical value of defecting from the collusive agreement in (14a) changes to
\[ \hat{\theta}^{OP}_{i} = \frac{[a_{j} - (1 + t_{j})c_{i}]^{2}}{a_{i}^{2} - 4[(1 + t_{j})c_{j} - (1 + t_{i})c_{i}]\{\alpha_{j} + a_{i} - (1 + t_{j})c_{j}\}}, \quad \text{(14a)} \]
whereas the critical value of the high-cost firm, \( \hat{\theta}^{OP}_{j} \), remains as in (14b).

It is straightforward to adapt eq. (15) in HS to the changes in (14a), whereas the changes in Case C are the same as under the destination principle. Lemma 2 in HS is thus unchanged, for the same reasons as discussed above for the destination principle.
2. Choice of Tax Principle

In the first scenario, which underlies Propositions 1 and 4, markets are of equal size and cost differences dominate tax rate differentials in both regimes. With this simplification, the corrected critical values for the low-cost firm 1 in (9a) and (14a) yield

\[ \bar{\theta}_{DP}^1 = \left( \frac{1 + t_1}{1 + t_2} \right) \left[ a - (1 + t_2)c_1 \right] = \frac{n_{DP}^1}{d_{DP}^1} \]  
\[ \bar{\theta}_{OP}^1 = \frac{\alpha_1^2}{\alpha_1^2 - 8\alpha_2[(1 + t_2)c_2 - (1 + t_1)c_1]} = \frac{n_{OP}^1}{d_{OP}^1} \]  

Equation (16)

To prove Proposition 1 in HS for the corrected analysis, we have to show that \( \text{sign} \ (t_1 - t_2) = \text{sign} \ [\bar{\theta}_{DP}^1 - \bar{\theta}_{OP}^1] \). A comparison of the numerators in (16) and (17) shows directly that \( \text{sign} \ (t_1 - t_2) = \text{sign} \ [n_{DP}^1 - n_{OP}^1] \). Hence a sufficient condition for Proposition 1 to hold is that the denominators in (16) and (17) satisfy \( \text{sign} \ (d_{DP}^1 - d_{OP}^1) = -\text{sign} \ (t_1 - t_2) = \text{sign} \ (t_2 - t_1) \). Defining \( \Delta = d_{DP}^1 - d_{OP}^1 \) gives

\[ \Delta = 8\alpha_2[(1 + t_2)c_2 - (1 + t_1)c_1] - 4(1 + t_1)(c_2 - c_1)[\alpha_2 + a - (1 + t_1)c_2] \]
\[ = 4c_2(t_2 - t_1)[2\alpha_2 - (1 + t_1)(c_2 - c_1)] \]

which confirms \( \text{sign} \ (t_2 - t_1) = \text{sign} \ \Delta \) from the two conditions in (7).

The second scenario, which underlies Propositions 2 and 5, focuses on isolated differences in tax rates, \( t_2 > t_1 \), whereas the market size parameters \( a \) and the firms’ unit costs \( c \) are identical. The corrections in this note affect the binding critical value under the origin principle \( (\bar{\theta}_{OP}^1) \), but not under the destination principle \( (\bar{\theta}_{DP}^1) \). Eq. (18) in HS thus changes to\(^1\)

\[ \bar{\theta}_{DP}^2 = \frac{(1 + t_2)}{(1 + t_1)} \frac{\alpha_1^2}{\alpha_2^2}, \quad \bar{\theta}_{OP}^1 = \frac{\alpha_1^2}{\alpha_2^2 [\alpha_2 - c(t_2 - t_1)]^2 - 4\alpha_2 c(t_2 - t_1)} \]  
\[ (18) \]

To prove Proposition 2 for the corrected analysis, we have to show that (see the appendix of HS)

\[ \frac{\partial \bar{\theta}_{DP}^2}{\partial a} - \frac{\partial \bar{\theta}_{OP}^1}{\partial a} > 0, \quad \frac{\partial \bar{\theta}_{DP}^2}{\partial c} - \frac{\partial \bar{\theta}_{OP}^1}{\partial c} < 0. \]  
\[ (A.3) \]

\(^{1}\)In the denominator of \( \bar{\theta}_{OP}^1 \) we use \( \alpha_1^2 = [\alpha_2 + (t_2 - t_1)c]^2 = [\alpha_2 - (t_2 - t_1)c]^2 + 4\alpha_2 c(t_2 - t_1) \).
the origin principle, but not under the destination principle:
\[
\frac{\partial \bar{\theta}^{DP}_{2}}{\partial a} = -\frac{2(1 + t_2)\alpha_1\alpha_2(t_2 - t_1)c}{(1 + t_1)\alpha_2^2} \equiv \frac{n^{DP}_{2}}{d^{DP}_{2}}
\]
\[
\frac{\partial \bar{\theta}^{OP}_{1}}{\partial a} = -\frac{8\alpha_1(t_2 - t_1)[\alpha_2 - (t_2 - t_1)c]c}{\{[\alpha_2 - c(t_2 - t_1)]^2 - 4\alpha_2c(t_2 - t_1)\}^2} \equiv \frac{n^{OP}_{2}}{d^{OP}_{2}}.
\]
Both numerators in (A.4) are negative\(^2\), but denominators are positive. Hence a sufficient condition for the first part of (A.3) to hold is that \(n^{DP}_{2} > n^{OP}_{2}\) and \(d^{DP}_{2} > d^{OP}_{2}\).

The latter condition is clearly satisfied from (A.4). To establish the former we introduce
\[
\Gamma \equiv n^{DP}_{2} - n^{OP}_{2}
\]
and recall that \(t_2 > t_1\). Then
\[
\Gamma = 8\alpha_1c(t_2 - t_1)\left[\alpha_2 - (t_2 - t_1)c\right] - 2(1 + t_2)\alpha_1\alpha_2c(t_2 - t_1)
\]
\[
= 2\alpha_1c(t_2 - t_1)\left[[2 - (1 + t_2)]\alpha_2 + 2[\alpha_2 - 2c(t_2 - t_1)]\right] > 0,
\]
since \(t_2 < 1\) and the last term in the curly bracket is positive (cf. footnote 2). \(\square\)

Partial differentiation with respect to \(c\) proceeds analogously. Equation (A.5) in HS changes to
\[
\frac{\partial \bar{\theta}^{DP}_{2}}{\partial c} = \frac{\partial \bar{\theta}^{DP}_{2}}{\partial a} \left(-\frac{a/c}{1}\right) = \frac{n^{DP}_{3}}{d^{DP}_{2}}, \quad \frac{\partial \bar{\theta}^{OP}_{1}}{\partial c} = \frac{\partial \bar{\theta}^{OP}_{1}}{\partial a} \left(-\frac{a/c}{1}\right) = \frac{n^{OP}_{3}}{d^{OP}_{2}}.
\]
Since both numerators are now positive and \(d^{DP}_{2} > d^{OP}_{2}\), a sufficient condition for the second part of (A.3) to hold is that \(n^{OP}_{3} - n^{DP}_{3} > 0\). But this follows from \(\Gamma > 0\). \(\square\)

Proposition 3 is unaffected by the corrections reported in this note. In the analysis of tax rate harmonization, Proposition 4 follows directly from differentiating the corrected equations (16) and (17) with respect to \(t_1\) and \(t_2\). Similarly, Proposition 5 follows from differentiating the corrected equation (18) with respect to the two tax rates.

Reference:


\(^2\)From the requirement that the denominator of \(\bar{\theta}^{OP}_{1}\) in (18) must be positive, it follows that \([\alpha_2 - c(t_2 - t_1)]^2 > [\alpha_2 - 2c(t_2 - t_1)]^2 > [\alpha_2 - c(t_2 - t_1)]^2 - 4\alpha_2c(t_2 - t_1) > 0\).