Does Collective Wage Bargaining Restore Efficiency in a Search Model with Large Firms?∗

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Abstract

Existing search and bargaining models of the labor market show that firms hire inefficiently many workers when production is concave. Introducing decentralized collective wage bargaining into this environment, we demonstrate that this factual deviation from the standard individual wage bargaining assumption offsets firms’ overhiring incentive. The union wage hike, however, drives a wedge between the workers’ outside option and the social opportunity costs of working. We find that equilibrium employment is inefficiently low, such that collective bargaining cannot act as a second-best institution.

Keywords: labor market search, multi-worker firms, overemployment, collective wage bargaining.

JEL: J30, J50, J41.

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1 Introduction

This paper analyzes the efficiency effects of firm-level collective wage bargaining in a random search economy with concave production. In this framework without unions, an influential result is that firms hire more workers than the neoclassical marginal-cost-equals-marginal-productivity paradigm suggests (Smith, 1999; Cahuc and Wasmer, 2001; Cahuc et al., 2008). This leads to an inefficient equilibrium allocation. Intuitively, firms overemploy because, with concave production, the individually bargained wage is decreasing in the firm’s employment level. Search frictions thus provide a “first-mover-advantage” to firms as their vacancy choice affects the environment for future wage negotiations.\(^1\)

A classical result on union wage bargaining, summarized e.g. in Boeri et al. (2001), is that unions artificially restrict labor supply to raise wages. Thus, one could have the idea that, by allocating bargaining power to workers, collective bargaining at the firm-level may offset the strategic hiring incentives. In this paper we ask: is collective wage bargaining capable of restoring efficiency in an economy where individual wage bargaining leads firms to overemploy?\(^2\)

The relevance of this question stems from the fact that in most western economies a dominant way of wage determination is some form of union wage bargaining (see e.g. Flanagan, 1999). This is true for continental Europe where the union wage coverage, i.e. the share of the labor force covered by collective bargaining agreements, regularly exceeds 60% depending on the sector and the country. Moreover, albeit to a lesser extent, collective wage bargaining is also important in Anglo-Saxon economies (CESifo DICE, 2009; Alesina et al., 2005). With union-friendly legislative bills like e.g. the Employee Free Choice Act introduced into Congress in 2009 under consideration, collective bargaining agreements may also gain importance in the United States.

\(^{1}\)Closely related, in a static environment, Stole and Zwiebel (1996a, 1996b) show how non-binding wage contracts and intra-firm bargaining may imply that workers are overhired to limit their bargaining power.

\(^{2}\)In static models with Stole-Zwiebel bargaining, collective wage bargaining removes the wage externality by hindering firms from instantaneous renegotiations (cf. Stole and Zwiebel, 1996b, Sec. III B). In a dynamic environment with labor market frictions, where the individually bargained wage rate reproduces the Stole-Zwiebel solution, the effect of employment on wages survives because employment is predetermined when wages are bargained. This is the crux in the search and bargaining economy: firms gain a strategic advantage in the bargain precisely because of the search friction.
Besides its real-world occurrence, studying the effects of collective bargaining in an environment with search frictions and large firms is important because this model is a standard building block in a number of labor market studies and related applications. However, the effects of different bargaining institutions in this framework are largely unexplored (cf. already Mortensen, 1992, 166 or, more recently, Rogerson and Shimer, 2010).

In our analysis we model collective wage bargaining as a binding commitment of a firm’s employees to bargain together for the same wage and decide jointly whether to work (Calmfors and Lang, 1995; Stole and Zwiebel, 1996b).

With this wage bargaining institution, we first prove that equilibrium employment is less than under individual bargaining. Hence, collective wage bargaining counteracts the strategic incentive for overemployment. Moreover, our main result shows that the equilibrium employment level is indeed inefficiently low. The reason for this is as follows. The Nash bargain between the firm and all its employees makes the firm’s fraction of the surplus independent of the number of workers. Therefore the firm chooses employment to maximize the surplus. Interestingly, this has the consequence that the firm behaves as if bargaining was over both employment and the wage. If there was no hold-up arising from the ex-ante investment in worker-search (cf. Acemoglu and Shimer, 1999), the firm-level employment-wage pair would thus lie on the contract curve. In an equilibrium framework, however, the workers’ outside option in the wage bargain is endogenous (cf. Layard and Nickell, 1990 for a similar reasoning). The union wage hike increases this value beyond the social opportunity

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3 E.g., Cahuc et al. (2008) adapt the large firm setting to an economy with heterogenous labor and show that the substitutability between types of labor affects overemployment. Mortensen (2009) demonstrates that the multi-worker model with diminishing returns and intra-firm bargaining fits observable patterns of wage and productivity dispersion well. Beugnot and Tidball (2010) study a variant of the standard model with monopolistic competition and, because of increasing returns in the aggregate production function, multiple equilibria.

4 Most recently, Ebell and Haefke (2009) embed the model in a monopolistically competitive environment and analyze the interaction between product market regulation and equilibrium unemployment. Felbermayr and Prat (2010) enrich the large-firm random search model with firms that differ in productivity to show the effect of product market regulation on firm selection.

5 Empirically, it is fair to say that collective wage bargaining occurs on several levels, including the national, sectoral, and the firm level. However, there is a clear tendency towards more decentralized forms of collective wage determination. This is true for countries where wage determination at the sectoral level (cf. Ochel, 2005 and Gürtzgen, 2009 for Germany) and at the national level traditionally played an important role. See Moene and Wallerstein (1997) for evidence and an analysis of the effects of the level of wage setting on wage compression.

6 This is in contrast to individual bargaining, where the firm’s fraction of the surplus is a function of the employment level. The firm then opens vacancies strategically to increase its fraction of the surplus.
costs of working, thereby causing equilibrium employment to fall below the efficient level.

Our paper relates to two strands of literature. First, we build on the work of Smith (1999) and Cahuc and Wasmer (2001). These papers were the first to analyze the possibility of overemployment within a (random) search model. Both papers showed, independently and within slightly different frameworks, that firms, when operating under decreasing marginal products, hire workers up to levels where the marginal product falls short of marginal costs. Two exceptions avoid this inefficiency in multi-worker-firm search models. The first is competitive search where firms are allowed to commit ex-ante to a wage. These models eliminate the inefficiency altogether, see Kaas and Kircher (2010) for a recent contribution. However, there is also no role to play for the type of wage setting regime. The second exception is provided by Cahuc and Wasmer (2001). They show how the existence of an additional factor of production (they consider capital) may remove the incentives for overemployment by effectively linearizing the production technology. This requires a constant returns to scale technology and that the additional factor is freely and instantaneously adjustable. However, none of these papers incorporates collective wage bargaining.

Second, the paper relates to the analysis of union wage determination in frictional labor markets. Following Pissarides (1986), a number of studies including Delacroix (2006) and Garibaldi and Violante (2004) shed light on the effects of unions in search models, showing in particular that collective wage bargaining raises equilibrium unemployment. These papers do not, however, consider the case where individual bargaining implies overemployment. This case is interesting because the necessary conditions (concave technologies, or downward sloping demand, and no ex-ante commitment to a wage rate) are probably relevant for many jobs. Bringing together the effects of different bargaining regimes and large firms therefore

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7 A very early contribution is Bertola and Caballero (1994). They also consider large firms in a model with search frictions and take the strategic/monopsonistic employment motives into account. Their focus, however, is on the employment and the wage path of firms that face productivity shocks (labor hoarding). They do not analyze the effect of different bargaining institutions.

8 For a notable exception see Michelacci and Suarez (2006) who combine competitive wage posting with ex-post wage bargaining. They show that although wage bargaining gives rise to a hold-up problem, firms may opt for this type of wage setting regime in order to condition wages on qualifications.

9 Mattesini and Rossi (2007), Zanetti (2007), Faia and Rossi (2009), and Morin (2010) study the effects of unions on wage volatility in frictional labor markets. In these models, linearity or timing assumptions ensure the absence of strategic employment decisions, which is the focus in this paper. For a classical survey on union wage setting, employment, and investment see e.g. Lever and van Veen (1991).
adds an important perspective on the functioning of real-world labor markets.

The plan of the paper is as follows. Section 2 lays out the environment and describes the timing of events. Section 3 characterizes the constrained efficient allocation. Section 4 derives the equilibrium under collective bargaining. Section 5 compares the equilibrium allocations under individual and collective bargaining and establishes efficiency properties. We conclude in Section 6 by discussing the contribution and avenues for further research.

2 General Setup

2.1 Environment

Consider a search and bargaining economy in continuous time. The model is populated by a unit mass of homogenous workers who each supply one unit of labor inelastically. There is a continuum of firms, each opening a continuum of vacancies and employing a continuum of workers (workers are negligibly small relative to firms). All agents are risk-neutral, infinitely-lived, anonymous, and discount future income at constant rate $r$. Firms are endowed with a homogenous production technology $F(N_i)$, $F' > 0 \geq F''$, $F(0) = 0$, and produce a unique consumption good which is the numéraire.

At every point in time, a firm’s workforce $N_i (\geq 0)$ is predetermined due to the search friction. A vacancy involves a flow cost $c > 0$. The aggregate number of matches between workers and firms per unit time is given by $M(U,V)$, where $M$ is the labor market matching function. $M$ is increasing in the aggregate number of vacancies $V$ and the pool of unemployed workers $U = 1 - N$, concave, and homogenous of degree 1. Matches are random so that, in each short time interval $dt$, a vacant position is filled with probability $\frac{M}{V} dt = M \left( \frac{U}{V}, 1 \right) dt \equiv \lambda_m(\theta) dt$, where $\lambda_m < 0$, $\eta(\theta) \equiv \lambda_m \frac{\theta}{\lambda_m} \in (-1, 0)$, and $\lim_{\theta \downarrow 0} \lambda_m = +\infty$. An increase in the ratio of total vacancies to searching workers, the tightness $\theta \equiv \frac{V}{U}$, lowers the rate at which hiring firms match with a worker. An unemployed worker will match with a firm with probability $p(\theta) \equiv \frac{M}{U} = \theta \lambda_m$, $p' > 0$.

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10 The time argument is suppressed unless this might cause confusion. Throughout the paper, we focus on steady states.

11 Following Cahuc et al. (2008) and Kaas and Kircher (2010), this allows us to apply a law of large numbers at the firm level.
While employed with firm $i$, a worker receives a flow income equal to the (continuously) bargained wage $w_i$. Occupied jobs are exogenously destroyed by job-specific i.i.d. shocks at Poisson rate $\lambda_i$. During search, workers enjoy an exogenous flow return $b$ ($\geq 0$), measured in terms of the numéraire.

Our view on worker unions is deliberately simplistic. In each firm, the group of employees bargains collectively for a uniform wage rate, subject to the same type of contracts as individual workers, and decides jointly whether to work. The goal of the union is to maximize the sum of the value of being employed over all employees in the firm. This focus allows us to draw a clear comparison to the hiring decisions and aggregate outcomes under the bargaining institution previously analyzed in the literature.\(^{12}\)

### 2.2 Timing and Incentives for Strategic Behavior

At every point in time, in line with the standard model, each employer first opens vacancies and then bargains wages with its current employees.\(^{13}\) When recruiting, the firm rationally anticipates that, if production (or demand) gives rise to a non-linear product of labor, the bargained wage is a function of its employment level.\(^{14}\) As a result, wage outcomes that decline in employment cause an incentive for over-hiring (and vice versa).

Below we study differences in hiring motives and ensuing equilibrium effects of collective and individual wage determination. To begin, we characterize the constrained planner solution, which is the efficiency benchmark.

### 3 Constrained Efficient Allocation: Social Planner

Consider a utilitarian planner who faces the same matching frictions as workers and firms. Aggregate employment increases by matches of workers and hiring firms and decreases by

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\(^{12}\)By focussing sharply on the difference in the wage bargaining institution, we abstract from a variety of important features, such as working conditions, commonly covered by collective bargaining agreements. We also ignore problems that may arise when workers’ preferences are heterogenous.

\(^{13}\)This timing assumption implies that vacancy costs are sunk when wages are bargained, so that the firm incurs these costs independent of whether or not the bargain is successful.

\(^{14}\)Here we model firms as price-takers and impose concavity on the production function. Alternative assumptions that imply curvature on the product of labor yield qualitatively similar results.
destroyed jobs:
\[ dN = \lambda_m(\theta) V dt - \lambda_s N dt. \] (1)

The planner chooses, subject to (1), a sequence of vacancies that maximizes the present value of income minus vacancy costs
\[ SW = \int_t^\infty e^{-r(\tau-t)} \left[ F(N) + (1 - N) b - cV \right] d\tau. \]

By standard arguments, the problem can be written recursively, with \( N \) as the only state (see e.g. Turnovsky, 2000). Making use of (1), the value function solves the Bellman equation
\[ rSW(N) = \max_V \{ F(N) + (1 - N) b - cV + SW'(N) [\lambda_m(\theta)V - \lambda_s N] \}. \] (2)

Optimality requires the expected search costs for the marginal worker, accounting for the congestion externality, to equal the increase in welfare due to the hire:
\[ SW'(N) = \frac{c}{\lambda_m(\theta) + \lambda'_m(\theta) \theta}. \] (3)

Solving the planner’s problem, we get the socially efficient number of jobs.\(^{15}\)

**Lemma 1 (Planner Solution: Smith, 1999)** Let \( \eta(\theta) \equiv \frac{\lambda'_m(\theta)}{\lambda_m(\theta)} \theta. \) In steady state, the planner’s policy function \( N(\theta) \) satisfies
\[ F'(N) = b + \frac{c}{\lambda_m(\theta)} \frac{r + \lambda_s - \lambda_m \theta \eta(\theta)}{1 + \eta(\theta)}. \] (4)

**Proof:** Appendix.

Together with the Beveridge Curve, i.e. combinations of \( \theta \) and \( N \) for which \( dN/dt = 0 \), (4) determines the unique stationary constrained efficient allocation \((N^P, \theta^P)\).

As is well-known, in the linear, or effectively linear, Mortensen-Pissarides models there exists a surplus sharing rule consistent with the generalized Nash bargaining solution to the wage bargain that decentralizes this allocation (Hosios 1990, Pissarides 1990). If firms may affect the wage at any bargaining weight by hiring workers strategically, however, imposing the Hosios condition does not restore efficiency (cf. Acemoglu and Shimer, 1999). In particular, if \( F'' < 0 \) and wages are bargained between individual workers and the firm, firms still

\(^{15}\)As the planner is indifferent concerning the allocation of the surpluses, the aggregate policy function is basically the same as in the linear random search model (cf. Pissarides, 2000 ch. 8).
hire an inefficiently large number of workers (Smith, 1999).

We now solve for the equilibrium with collective wage bargaining. The following sections then compare individual and collective bargaining and derive efficiency properties.

4 Equilibrium with Collective Wage Bargaining

We focus on symmetric Markov search equilibria in steady state, where strategies depend exclusively on payoff relevant state variables, employment and wages are the same in all firms, and aggregate employment and the market tightness are constant over time. By search equilibrium we mean a vector \((N, \theta, w)\) that simultaneously solves the firms’ forward-looking hiring decisions and wages that support the bargaining outcomes.

4.1 Job Creation

Each firm chooses a path for vacancies to maximize the present value of its profit flow
\[
\Pi_i = \int_t^\infty e^{-r(\tau-t)} \pi_i(\tau) d\tau,
\]
taking as given the tightness of the labor market and the evolution of its workforce,
\[
dN_i = \lambda_m(\theta)V_i dt - \lambda_s N_i dt, \quad N_i \geq 0.
\]

In calculating instantaneous profits, \(\pi_i = F(N_i) - w_i(N_i)N_i - cV_i\) (revenues minus wage and vacancy costs), the firm anticipates the effect of employment on bargained wages.

As the firm’s problem is stationary, we solve it recursively. Making use of (5), \(\Pi_i\) satisfies the Bellman equation
\[
r\Pi_i(N_i) = \max_{V_i}\{F(N_i) - w_i(N_i)N_i - cV_i + \Pi_i'(N_i)[\lambda_m(\theta)V_i - \lambda_s N_i]\}.
\]

The right-hand side describes the firm’s optimal vacancy choice, which is to set the marginal value of employment equal to the expected search costs:
\[
\Pi_i'(N_i) = \frac{c}{\lambda_m(\theta)}.
\]

Differentiating the maximized Bellman equation with respect to \(N_i\), using the envelope
condition, and the law of motion of \( N_i \), we obtain the value of a marginal job in steady state:

\[
\Pi_i' (N_i) = \frac{F' (N_i) - w_i (N_i) - w_i' (N_i) N_i}{r + \lambda_s}.
\]  

(8)

Hiring an additional worker raises revenues by \( F' \), adds \( w_i \) to the wage bill, and affects the bargained wage of all employees, until the job is destroyed. Concave production implies that the externality on existing wages is negative \( (w_i' < 0) \), thus raising the worker’s value to the firm. In a sense, the matching constraints allow the firm to exert monopsonistic power, because its vacancy choice predetermines employment at the time of the wage bargain.\(^{16}\)

Combining (7) and (8) yields a first expression for the firm’s job creation curve:

\[
F' (N_i) - w_i' (N_i) N_i = w_i (N_i) + (r + \lambda_s) \frac{c}{\lambda_m (\theta)}.
\]  

(9)

Workers are hired up to the point where the marginal return from production and savings on existing wages equals the sum of wage and periodized search costs.\(^{17}\)

The key feature that distinguishes the job creation/labor demand condition in (9) from static right-to-manage models is the dependence on the wage setting curve, which is derived in the next section.

### 4.2 Value of a Match and Wage Determination

Wages are bargained bilaterally between the firm and its workers (for ease of interpretation, we can think of a representative worker who carries out the negotiation).\(^{18}\) Following Bertola and Caballero (1994) and the tradition of the seminal search literature (cf. Diamond, 1981, 1982; Mortensen, 1978, 1982; Pissarides, 1985, 2000), we adopt the generalized Nash bargaining solution\(^{19}\)

\[
\max_{w_i} \left( N_i r \bar{W_i} \right)^\beta \left( r \bar{\Pi_i} \right)^{1-\beta}
\]

(10)

\(^{16}\)Note the difference to standard monopsony models (see e.g. Manning, 2003) in which it is usually assumed that \( w' > 0 \) due to a positively sloped labor supply curve.

\(^{17}\)Of course, if marginal products are constant, whereby \( w_i' = 0 \), (9) will be standard Mortensen-Pissarides (which for \( \lambda_m \to \infty \) converges to the Walrasian \( F' = w \)).

\(^{18}\)The workers’ representative’s surplus then is equal to the sum of all employees’ surpluses.

\(^{19}\)Due to the continuous time framework, there will be continuous wage bargaining. Hence, it seems natural to assume that the bargaining parties split the “instantaneous pie” and consider flow values.
where $\beta \in (0, 1)$ is the bargaining power of the workers’ representative, while $N_i r \bar{W}_i$ and $r \bar{\Pi}_i$ are the union’s and the firm’s surplus, respectively.\footnote{Alternative configurations consistent with a strategic bargaining model, that e.g. considers the collective decision whether to work, yield similar predictions. The representation above matches the reference case with individual bargaining, which is briefly sketched below, as close as possible. More research into alternative bargaining protocols (including endogenous search effort, counteroffers, and wage-tenure contracts) is certainly important to put policy conclusions on solid grounds.}

A worker’s surplus is standard. With $z \in \{w, b\}$ denoting the instantaneous income (workers cannot save) and the corresponding employment state, her lifetime utility is $W^z = E_t \int_t^\infty e^{-r(\tau-t)} z d\tau.\footnote{Expectations are formed with respect to future income. Although employment at the firm level follows a deterministic process, the labor market state of one individual worker evolves stochastically (the firm knows that it will be separated from some workers, but not from whom).}$ When working for firm $i$ at a constant wage, $W^w_i$ supports the Bellman equation

$$r W^w_i = w_i - \lambda_s \bar{W}^w_i. \tag{11}$$

The worker earns $w_i$ and suffers a capital loss $\bar{W}^w_i \equiv W^w_i - W^b$ in case she loses her job. The value of working equals the appropriately discounted difference between the wage income and the flow value of search, which firms and workers take as given:

$$\bar{W}_i = \frac{w_i - r W^b}{r + \lambda} = \frac{w_i}{r + \lambda}. \tag{12}$$

For the firm, as vacancies are opened before wages are determined, the surplus from reaching a bargaining agreement is equal to the flow value of production:

$$r \bar{\Pi}_i (N_i) = F (N_i) - w_i (N_i) N_i. \tag{13}$$

Combining the first order maximization condition derived from (10) with (12) and (13), we obtain the wage rate as weighted average of the value of being unemployed and the average product:

$$w_i^{WS} = (1 - \beta) r W^b + \beta \frac{F (N_i)}{N_i}. \tag{WS}$$

The wage setting condition is reminiscent of static models with non-binding wage contracts. There, under individual bargaining, firms may exploit, by means of renegotiations, concave production technologies to lower wages. In such an environment, collective bargaining pre-
vents renegotiations with individual workers and hence effectively “destroys” this strategic incentive (cf. Stole and Zwiebel, 1996b, Sec. III.B). In our framework, however, search frictions in the labor market give the firm a competitive edge, even when wages are bargained collectively. This is because employment is endogenous and predetermined when wages are bargained.

With concave revenues, the bargaining outcome in (WS) implies that the firm may lower the uniform wage rate by opening vacancies strategically:

\[
\delta^C(N_i) \equiv \frac{dw^W_i}{dN_i} = \frac{\beta}{N_i} \left[ F'(N_i) - \frac{F(N_i)}{N_i} \right] (< 0). \tag{14}
\]

This wage externality completes the firm’s job creation condition. Combining (9) and (14), the optimal employment level satisfies

\[
w^{JC}_i = (1 - \beta) F'(N_i) + \beta \frac{F(N_i)}{N_i} - (r + \lambda_s) \frac{c}{\lambda_m(\theta)}. \tag{JC}
\]

Interestingly, if there was no search cost hold-up (i.e. with \(c = 0\)), equations (WS) and (JC) resemble the optimality conditions in the static neoclassical framework of McDonald and Solow (1981), where firms and unions bargain over both employment and the wage rate.22 The corollary is that, although firms in our framework choose employment unilaterally, the employment-wage pair would then lie on the contract curve. This is due to the Pareto-optimality of the Nash bargain between the firm and the union, which ensures that the firm’s fraction of the surplus is constant. Hence, in contrast to the individual bargaining benchmark (where the firm’s fraction is an increasing function of employment), the firm’s optimal behavior is to maximize the surplus.

(WS) and (JC) determine employment and wages at the firm level. Employment satisfies

\[
F'(N_{iC}) = rW^b + \frac{r + \lambda_s}{1 - \beta} \frac{c}{\lambda_m(\theta)}. \tag{15}
\]

Equation (15) reflects the standard search cost hold-up (the periodized search cost is multiplied by \(\frac{1}{1-\beta}\)). Absent this distortion, however, hiring is “efficient” in the sense that

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22In particular, (JC) is their equation (8) for \(c = 0\).
the marginal production value of a worker equals the worker’s outside option. As such, collective wage bargaining offsets the hiring externality resulting from individual bargaining. Efficiency may not prevail in equilibrium because the worker’s outside option is endogenous and, as shown below, will differ from the social opportunity costs of working.

To characterize the wage, we define the production elasticity \( \alpha(N) \equiv F'N^\alpha \) that relates average and marginal product:

\[
w_i^C = (1 - \beta) rW^b + \frac{\beta}{\alpha(N_i^C)} \left[ rW^b + \frac{r + \lambda_s c}{1 - \beta \lambda_m(\theta)} \right].
\]

Employment (the wage rate) is declining (increasing) in expected search costs, the value of being unemployed, and the workers’ bargaining power. A new feature of the model is that the bargained wage is increasing in the curvature of the revenue function as inversely measured by \( \alpha \). If \( \alpha' = 0 \), employment is decreasing in the curvature of \( F \) (cf. the discussion after Lemma 4 below).

We now consider the equilibrium allocation.

### 4.3 Labor Market Equilibrium

In the stationary equilibrium, wages and employment are constant and identical in all firms. With \( dN = 0 \), the law of motion for \( N \) yields a Beverdige curve:

\[
(1 - N) \theta \lambda_m(\theta) - N \lambda_s = 0.
\]

By standard arguments, the value of being unemployed solves

\[
rW^b = b + p(\theta) (W^w - W^b).
\]
The equilibrium \( rW^b \) then is an increasing function of \( w \) and \( \theta \) that averages incomes in employment and unemployment spells:\(^{23}\)

\[
rW^b = \left( \frac{r + \lambda_s}{r + \lambda_s + p(\theta)} \right)^b + \frac{p(\theta)}{r + \lambda_s + p(\theta)}w. \quad (19)
\]

**Definition 1 (Stationary Equilibrium)** A symmetric steady state equilibrium is a path of constant \( w, N, \theta, \) and \( rW^b \) that satisfies (WS), (JC), (17), and (19).

Combining (WS), (JC), and (19) we derive an aggregate policy function:

\[
F'(N) \left\{ 1 - \beta \frac{p(\theta)}{r + \lambda_s} \left[ \frac{1}{\alpha(N)} - 1 \right] \right\} = b + \frac{r + \lambda_s + \beta p(\theta) c}{1 - \beta} \lambda_m(\theta). \quad (20)
\]

Together with the Beveride Curve, (20) determines the equilibrium pair \((N^C, \theta^C)\).

**Proposition 1 (Existence, Uniqueness)** An equilibrium exists. For functional forms and parameters that guarantee stability, the equilibrium is unique.

**Proof:** Appendix.

5 **Collective vs. Individual Wage Bargaining**

5.1 **Benchmark: Individual Bargaining**

In analyzing the equilibrium, we make use of the following Lemma regarding equilibrium under individual wage bargaining (Smith, 1999; Cahuc and Wasmer, 2001; Cahuc et al., 2008). Part \((i)\) states the outcomes of decisions at the firm level. The assumption underlying the sharing rule is that wages are simultaneously bargained between the firm and each worker, allowing the use of a representative firm-worker pair.\(^{24}\) The surpluses are: for a worker, the additional value of working in equilibrium, derived from (11) and (18) and using symmetry, is

\[
W^w - W^b = \frac{w^w - b}{r + \lambda_s + p(\theta)}. \quad \text{Combining this and (18) gives (19).}
\]

\(^{23}\)The additional value of working in equilibrium, derived from (11) and (18) and using symmetry, is

\[
W^w - W^b = \frac{W^w - W^b}{r + \lambda_s + p(\theta)}. \quad \text{Combining this and (18) gives (19).}
\]

\(^{24}\)See Smith (1999), Cahuc and Wasmer (2001), and Cahuc et al. (2008) for more details. Although bilateral Nash bargaining is a natural choice due to the quasi-rents, simultaneous bilateral bargaining may be perceived as too simple an assumption that may be replaced by multilateral bargaining using the Shapley value. Note, however, that the standard Nash sharing rule replicates the wage schedule obtained under Stole-Zwiebel bargaining in the static model, which is a useful benchmark. Moreover, despite potential caveats, simultaneous bilateral bargaining allows us to apply the envelope condition to derive the firm’s net return
net value of employment $r(W^w - W^b)$; for the firm, the value of producing without the
marginal worker $r\Pi'_i$.\footnote{In the absence of a canonical micro-foundation for $\beta$, we set an individual worker’s bargaining power equal to the worker representative’s bargaining power.} Part (ii) states the policy function that together with the Beveridge Curve characterizes equilibrium with individual bargaining.

**Lemma 2 (Individual Bargaining: Smith, 1999; Cahuc and Wasmer, 2001)**

(i) In the stationary equilibrium with individual wage bargaining, given aggregates,

- the wage setting curve is given by

$$w_i^{WS,I} = [1 - \beta] rW^b + N_i \int_0^{N_i} x^{-\frac{1}{\beta}} F''(x) dx = [1 - \beta] rW^b + \int_0^1 x^{-\frac{1}{\beta}} F''(N_i x) dx, \tag{21}$$

- the job creation curve is given by

$$w_i^{JC,I} = F'(N_i) - N_i \int_0^1 x^{\frac{1}{\beta}} F''(N_i x) dx - \frac{r + \lambda_s}{\lambda_m(\theta)} c. \tag{22}$$

(ii) The aggregate policy function implied by (21) and (22) satisfies

$$\frac{1}{\beta} \int_0^1 x^{\frac{1}{\beta}-1} F'(N x) dx = b + \frac{r + \lambda_s + \beta p(\theta)}{1 - \beta} \frac{c}{\lambda_m(\theta)}. \tag{23}$$

(iii) Equilibrium exists and is unique.

**Proof:** Appendix.

The bargained wage reflects the worker’s outside option and (in contrast to collective bargaining) a weighted average of marginal productivities. In analogy to (WS), it is decreasing in employment. Hence, optimal job creation again takes an externality on wages into account ($-N_i \int_0^1 x^{\frac{1}{\beta}} F''(N_i x) dx > 0$).

For the following comparison, we conclude this section by solving for firm-level employment and wages under individual bargaining. Combining (21) and (22) characterizes from a match ($\Pi'$ is in fact the added value of any worker in each bargain). This is generally not the case in the presence of immediate renegotiations (cf. Acemoglu and Hawkins, 2010).
employment:

\[ F'(N_i^t) - N_i^t \int_0^1 x^{\frac{1}{\beta}} F''(N_i^t x) \, dx = rW^b + \frac{r + \lambda s}{1 - \beta} \frac{c}{\lambda_m(\theta)}. \]  \hspace{1cm} (24)

In contrast to collective bargaining, the strategic behavior of firms here drives a wedge between the net marginal value of a job and the private opportunity costs of working. This wedge implies that employment will be inefficiently large.

The corresponding wage, obtained from (22) and (24), satisfies

\[ w_i^t = rW^b + \frac{\beta}{1 - \beta} (r + \lambda s) \frac{c}{\lambda_m(\theta)}. \]  \hspace{1cm} (25)

Workers receive the reservation value \( rW^b \) and a fraction \( \frac{\beta}{1 - \beta} \) of the appropriately discounted savings on hiring costs due to the match.

5.2 Labor Market Outcomes

We now compare the labor market outcomes under the different bargaining regimes (at the firm level and in equilibrium). To begin, we consider decisions at the firm level, starting with the size of the wage externality.

Under individual bargaining, the wage externality from employment, derived from (21), is equal to

\[ \delta^I = \left[ -\frac{1}{\beta} N_i^{-\frac{2}{\beta}} \int_0^{N_i} x^{\frac{1-\beta}{\beta}} F'(x) \, dx + F'(N) \right] N_i^{\frac{1}{\beta}} = \int_0^1 x^{\frac{1}{\beta}} F''(N_i x) \, dx \quad (< 0). \]  \hspace{1cm} (26)

Comparing \( \delta^I \) and \( \delta^C \) under collective bargaining we find the following.

**Lemma 3 (Size of Wage Externality)** Suppose the production elasticity is constant \( (\alpha' = 0) \). At any given employment level, collective wage bargaining induces a larger wage externality than individual bargaining.

**Proof:** Appendix.

\(^{26}\)The second equality uses integration by parts.
In general, the wage declines in employment if the labor revenue function is subject to diminishing returns. Concavity implies that the average product of labor falls by more (in absolute terms) than the marginal product if employment increases.\textsuperscript{27} As collective bargaining shifts negotiations into more curved regions of the labor revenue function, savings on existing wages increase. Ceteris paribus, this increases the incentive for opening vacancies strategically.\textsuperscript{28}

In summary, with diminishing returns to the product of labor, collective bargaining exerts two countervailing effects on employment. On the one hand, relative to individual bargaining, firms have an incentive to increase labor demand because the strategic gain from hiring an additional worker is larger. On the other hand, the employment incentive deteriorates as the threat point of collectively bargaining workers is larger, allowing them to acquire a higher wage.

We next compare employment and wages under collective and individual wage bargaining for a given identical outside option and market tightness.

**Lemma 4 (Employment and Wages at the Firm Level)** Suppose $F'' < 0$ and take as given a labor market tightness, $\theta$, and a value of being unemployed, $rW^b$. Then, firm-level employment under collective bargaining is less than under individual bargaining ($N_C(rW^b, \theta) < N_I(rW^b, \theta)$) and the union wage exceeds the individually bargained wage ($w_i(rW^b, \theta) < w_i^C(rW^b, \theta)$).

**Proof:** Appendix.

Because the firm chooses employment such that the surplus is maximized, there is no overemployment. Firm-level wages are larger due to the better bargaining position of the workers.

Given the value of being unemployed, the wage under individual bargaining does not depend on the curvature of the production function.\textsuperscript{29} Under collective bargaining, however,

\textsuperscript{27} For a proof see the Appendix (proof to Lemma 3).
\textsuperscript{28} Note also that, as neither wage externality depends directly on the workers’ outside option, the strategic component at a given employment level is not affected by the prevailing state of the labor market.
\textsuperscript{29} Changing the curvature of the production function affects employment, which is the essence of the strategic behavior, but this affects both wage setting and job creation curves such that the firm-level wage remains unchanged.
concavity ($\alpha < 1$) allows the workers to bid up the wage. This is similar to static neoclassical models with unionization, where the curvature of the production function is a measure of the trade-off between wages and employment (see e.g. Layard et al., 2005). In this dynamic framework, a larger curvature further implies larger savings on existing wages, which outward-shifts the job creation curve, and raises wages.

To complete the solution of the model, we need to solve for the equilibrium tightness and employment using the Beveridge Curve and the aggregate policy function. While an explicit solution requires a specification for $\lambda_m$, such a solution is not necessary to characterize differences in the allocation under both bargaining regimes. Because the Beveridge Curve does not depend on the bargaining regime, it suffices to compare the policy functions.

**Proposition 2 (Equilibrium Allocation under both Bargaining Regimes)** Suppose $F'' < 0$. Equilibrium employment under collective bargaining is less than under individual bargaining ($N^C < N^I$). The same is true for labor market tightness ($\theta^C < \theta^I$).

**Proof:** Appendix.

Intuitively, starting from an equilibrium under collective bargaining and switching to individual bargaining, firms on impact have an incentive to open more vacancies. Consequently, for a given employment level, $\theta$ jumps up. This implies that inflows into employment exceed outflows. The economy then moves along the new policy function towards the new stationary equilibrium with larger employment and labor market tightness.

The equilibrium wage rate is obtained from (19) and (25) under individual bargaining,

$$w^I = b + \frac{\beta}{1 - \beta \lambda_m (\theta^I)} \left[ r + \lambda_s + p (\theta^I) \right],$$

and (16) and (19), respectively, under collective bargaining,

$$w^C = \frac{1 + \beta \lambda (N^C)}{1 - \beta \lambda (N^C) \frac{p (\theta^C)}{r + \lambda_s}} b + \frac{1}{\alpha (N^C)} \frac{\beta}{1 - \beta \lambda_m (\theta^C)} \left[ r + \lambda_s + p (\theta^C) \right] \frac{1}{1 - \beta \lambda (N^C) \frac{p (\theta^C)}{r + \lambda_s}},$$

where $\lambda (N) \equiv \frac{1}{\alpha (N)} - 1$.

Comparing the wage outcomes under both bargaining regimes, we find that (for a given $\theta$) unionized workers get a mark-up on the income of the unemployed and larger shares of
the search costs. But this wage hike is dampened by the decrease in the reemployment probability \( (\theta^C < \theta^I \text{ whereby } p(\theta^C) < p(\theta^I) ) \). Note that without the search cost hold-up, the wage under individual bargaining is equal to the social opportunity costs of working, whereas the collectively bargained wage exceeds these costs.

Up to this point, we have compared equilibrium allocations under both bargaining regimes. We now turn to our main question, namely whether, given the inefficient hiring decisions under individual bargaining, collective wage formation is some sort of second-best institution.

5.3 Efficiency

Because equilibrium employment and labor market tightness were characterized implicitly, we apply an indirect approach to analyze the efficiency properties of the two bargaining regimes. To this end we compare the policy functions under individual and collective bargaining, repeated here in (30) and (31) for convenience, to the one chosen by a social planner, repeated in (29):

\[
F'(N) = b + \frac{c}{\lambda_m(\theta)} \left( r + \lambda_s - p(\theta) \eta(\theta) \right) 1 + \eta(\theta),
\]

\[
F'(N) + \left[ \frac{1}{\beta} \int_0^1 x^{1-1} F'(N x) dx - F'(N) \right] = b + \frac{c}{\lambda_m(\theta)} \frac{r + \lambda_s + \beta p(\theta)}{1 - \beta},
\]

\[
F'(N) - \frac{\beta p(\theta)}{r + \lambda_s} \left[ \frac{F(N)}{N} - F'(N) \right] = b + \frac{c}{\lambda_m(\theta)} \frac{r + \lambda_s + \beta p(\theta)}{1 - \beta}.
\]

Let us impose the well-known Hosios (1990) condition, i.e. \(-\eta(\theta) = \beta\), so that the search cost hold-up offsets the congestion externality from search. In this case, the right-hand sides of (29)–(31) are identical. Both decentral allocations, however, are also characterized by inefficiencies arising from the bargaining process and strategic employment incentives; neither individual bargaining (cf. Smith, 1999) nor collective bargaining yields efficiency. Comparing the three policy functions leads us to the main result.

**Proposition 3 (Efficiency)** Suppose the Hosios condition holds and \( F'' < 0 \). Then, equilibrium employment under collective bargaining is inefficiently low, while employment under individual bargaining is inefficiently high \((N^C < N^P < N^I)\). Moreover, the labor market
tightness is inefficiently low under collective bargaining, while it is inefficiently high under individual bargaining ($\theta^I > \theta^P > \theta^C$). If $F'' = 0$, the equilibrium allocations under both bargaining regimes coincide and are equal to the efficient allocation.

**Proof:** Appendix.

Independent of the bargaining regime, the firm’s employment choice determines the size of the surplus (which is a function of output) that can be divided between the firm and its employees. Under individual bargaining, the hiring decision further affects the division of the surplus. This is because the wage externality makes the wage sum a function of weighted marginal products. Concave production implies that the employees’ fraction of the surplus is decreasing in employment, causing the firm to overemploy. As the workers’ outside option in the bargain correctly reflects the social opportunity costs of working (the usual hold-up and search externality are switched off using the Hosios condition), there is no counteracting equilibrium effect, and overemployment prevails at the aggregate level.

In contrast, under collective bargaining, the hiring decision does not affect the employees’ fraction of the surplus. Because the firm bargains with the union, the wage sum is a function of output, so that the ratio of wage sum and surplus is constant. Thus, the firm chooses employment to maximize the size of the surplus. At the firm level, if there was no search cost hold-up, the employment-wage pair would lie on the contract curve. However, the economy-wide union wage hike drives a wedge between the social opportunity costs of working and the workers’ outside option in the wage bargain. Thus, equilibrium employment is inefficiently low.

Note that the hiring decision only affects the division of the surplus if there is a wage externality that is not internalized by a union. If the production function (in fact, the revenue function) is linear in employment, there is no wage externality. The allocations under both bargaining regimes are then observationally equivalent, but for different reasons. Under individual bargaining, the firm has no incentive to open vacancies strategically.

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30 At the aggregate level, the Hosios condition offsets this hold-up (i.e., the right-hand sides of (29)–(31) are then equal irrespective of the search costs, $c$).

31 To avoid this feature, a number of union wage setting models with frictional labor markets conveniently assume that wages are bargained before workers arrive at the firm. Consistent with the nature of many wage contracts, we do not presume this level of contractual completeness in our search and bargaining economy.
collective bargaining, the unions cannot distort the workers’ outside option.

We discuss some implications of these findings after a short classification.

6 Conclusion

Collective wage bargaining is usually associated with an inefficient allocation in the labor market. The strategic advantage of deciding jointly whether to work allows employees to bargain higher wages by means of rationing labor, see e.g. McDonald and Solow (1981). This view is supported by standard neoclassical models in which collective bargaining is the only distortion in the labor market. More recently, it has become widely acknowledged that the labor market is characterized by a variety of important frictions. First and foremost, the fact that heterogeneity and information frictions prevent firms and workers from matching instantaneously has become a key element in now standard modeling frameworks (see e.g. Pissarides, 2000 or Rogerson et al., 2005). In this paper, we analyze the interaction of search frictions and the distortion caused by collective wage bargaining.

Building on the work of Smith (1999) and Cahuc and Wasmer (2001), we allow for non-linear products of labor. This case is interesting because it gives rise to strategic overemployment under individual bargaining. Whether this result survives with collective bargaining is not obvious. The possibility of going on strike allows unionized workers to bargain for higher wages. The firm, however, anticipates the bargaining outcome and chooses employment strategically. Comparing the effects on wage setting and job creation, we derive two main results. Employment under collective bargaining always falls short of employment under individual bargaining and is inefficiently low.

Intuitively, the Nash bargain with all employees internalizes the wage externality and forces the firm to choose employment such that the surplus is maximized. This firm-level result, however, does not carry over to the aggregate level because the union wage is too high in the sense that the workers’ outside option in the wage bargain exceeds the social opportunity costs of working.

These insights have important policy relevance. First, in labor markets where individual bargaining leads firms to overemploy, collective bargaining cannot act as a second-best in-
stitutional setting. Second, the welfare implications of policies that directly affect the wage externality (e.g., minimum wages, regulation of the length of contracts) depend on the bargaining regime. Under individual bargaining, reducing the wage externality increases welfare. Under collective bargaining, doing so distorts the hiring decision while the wedge between the union wage and the social costs of working remains unaffected, thus reducing welfare. The general message is that optimal labor market policies have to take the bargaining regime into account.

Our framework offers a number of avenues for fruitful further research. First, it is important to study optimal wage flexibility under different bargaining institutions to get deeper insights into the interactions of various frictions in the labor market. Second, beyond steady state solutions, differences in bargaining institutions may have pronounced implications for the dynamic behavior of wages and employment over the business cycle. A dynamic analysis may help reconcile the different labor market experience of continental Europe vis-à-vis the U.S. (see e.g. Hall, 2003 and Shimer, 2004 for an analysis of the effects of wage rigidity under individual bargaining). Finally, the interaction of worker heterogeneity and the bargaining regime is relevant for understanding real world labor markets. Boeri and Burda (2009) study collective bargaining (and union formation) with heterogeneous skills and linear production. Linearity, however, suppresses strategic interactions within the firm. Our model may serve as basis for analyzing how wage externalities affect the skill composition within the firm, the degree of unionization, and equilibrium unemployment.
References


A Appendix

Proof of Lemma 1: Planner Solution.
Differentiating the maximized Bellman equation with respect to $N$ gives a differential equation for the evolution of the co-state variable $SW'$,

$$rSW''(N) = F'(N) - b - c\frac{dV}{dN} + SW''(N) [\lambda_m(\theta)V - \lambda_sN]$$

$$+ SW' \left\{ [\lambda_m(\theta)\theta + \lambda_m(\theta)] \frac{dV}{dN} - \lambda'(\theta)\theta^2 \frac{dU}{dN} - \lambda_s \right\}.$$

In steady state, $\frac{dN}{dt} = 0$. Substituting with (3) and noting that $\frac{dU}{dN} = -1$ by definition gives the solution. ■

Proof of Proposition 1: Existence, Uniqueness.
The policy function combines the reservation wage equation with the job creation and wage setting curves. For given $rW^b$, the firm’s job creation curve is steeper in absolute terms than the wage setting curve, ensuring stability of the solution.\(^{32}\) If $rW^b$ is endogenous, substituting with (19) shows that the equilibrium feedback effect on wage setting must not be too large to ensure stability (recall that $rW^b(w, \theta)$ enters the wage equation with factor $1 + \beta(\frac{1}{\alpha} - 1)$). Economically, the marginal product must not fall too fast relative to the average product. This guarantees that savings on existing wages fall short of the firm’s willingness to pay for an additional worker in absolute terms:

$$\left| \frac{dw^{WS}}{dN} \right| < \left| \frac{dw^{JC}}{dN} \right|. \quad (A-1)$$

Throughout, we focus on functional forms and parameters that satisfy (A-1).

\(^{32}\) As a short-cut to the fully dynamic argument, consider comparative statics in a stationary equilibrium with a job creation curve that is less steep (in absolute terms) than the wage setting curve in $(w, N)$-space. Adding a worker would reduce the negotiated wage to a level where the firm is willing to hire more workers, hence bringing the economy farther away from the equilibrium, implying instability. In contrast, if the wage setting curve is less steep than the job creation curve, adding a worker reduces the wage only up to a point where the firm is not willing to keep the worker, thus returning to equilibrium.
We now rewrite (20) using the definition of \( \alpha \)

\[
\Gamma_C (N, \theta) \equiv \frac{\beta p(\theta)}{r + \lambda_s} \left[ \frac{F(N)}{N} - F'(N) \right] - F'(N) + b + \frac{c}{\lambda_m(\theta)} \frac{r + \lambda_s + \beta p(\theta)}{1 - \beta} = 0. \quad (A-2)
\]

Implicit differentiation shows that \( N'(\theta) < 0 \) if and only if \( \partial \Gamma_C (N, \theta) / \partial N > 0 \) because \( \partial \Gamma_C (N, \theta) / \partial \theta > 0 \). Note that the sign of \( \partial \Gamma_C (N, \theta) / \partial N \) is equal to the sign of

\[
-\frac{\beta p(\theta)}{N} \left[ \frac{F(N)}{N} - F'(N) \right] - \left[ r + \lambda_s + \beta p(\theta) \right] F''(N)
\]

so that

\[
\frac{\partial \Gamma_C (N, \theta)}{\partial N} > 0 \Leftrightarrow \frac{NF''(N)}{F'(N) - \frac{F(N)}{N}} > 0 \Leftrightarrow \frac{\beta p(\theta)}{r + \lambda_s + \beta p(\theta)}.
\]

(A-3)

Combining (WS) and (19) implies:

\[
dw^{WS} = (1 - \beta) r \left[ \frac{\partial W^b}{\partial w} dw + \frac{\partial W^b}{\partial \theta} d\theta \right] + \beta \left[ \frac{F(N)}{N} \right]' dN
\]

\[
\Leftrightarrow \frac{dw^{WS}}{dN} = \frac{\beta \left[ \frac{F(N)}{N} \right]'}{1 - (1 - \beta) \frac{\partial (rW^b)}{\partial w}} = \beta \frac{r + \lambda^s + p(\theta)}{r + \lambda^s + \beta p(\theta)} \left[ \frac{F(N)}{N} \right]'.
\]

(A-4)

Similarly, combining (JC) and (19) implies:

\[
\frac{dw^{JC}}{dN} = (1 - \beta) F''(N) + \beta \left[ \frac{F(N)}{N} \right]'.
\]

(A-5)

From (A-1), (A-4), and (A-5) we thus have

\[
-\beta \frac{r + \lambda^s + p(\theta)}{r + \lambda^s + \beta p(\theta)} \left[ \frac{F(N)}{N} \right]' < -(1 - \beta) F'' - \beta \left[ \frac{F(N)}{N} \right]'
\]

\[
\Leftrightarrow -\frac{\beta p(\theta)}{r + \lambda^s + \beta p(\theta)} \left[ \frac{F(N)}{N} \right]' < -F''.
\]

(A-6)

Substituting with \( [F(N)/N]' = 1/N[F' - F/N] \), the following holds in a stable equilibrium:

\[
\frac{\beta p(\theta)}{r + \lambda^s + \beta p(\theta)} < \frac{-F(N)'' N}{F(N) - F'(N)} = \alpha(N) - \alpha(N)'N.
\]

(A-7)
Together with (A-3), this implies $N' (\theta) < 0$. As $\Gamma_C (N_0, 0) = 0$ also yields $F' (N_0) = b$, and hence $N_0 > 0$, a unique (intersection with the Beveridge Curve and hence a unique) steady state exists.

**Proof of Lemma 2: Equilibrium with Individual Wage Bargaining.**

**Part (i):** The bargained wage is given by

$$ w_i = \arg \max \left\{ \left( r \bar{W}_i \right)^{\beta} (r \Pi'_i)^{1-\beta} \right\}. \quad (A-8) $$

Using (8) and (12), the first-order maximization condition derived from (A-8) satisfies the linear first-order differential equation

$$ w_i = [1 - \beta] r W^b + \beta [F' (N) - w'_i (N) N]. \quad (A-9) $$

The particular solution in (21) is obtained, as in Cahuc and Wasmer (2001) and Cahuc et al. (2008), assuming the boundary condition $\lim_{N_i \to 0} N_i w_i (N_i) = 0$. Making use of integration by parts to combine (26) and (9) gives (22).

**Part (ii):** Combining the reservation wage equation (19) with the firm-level wage (25) gives the reservation wage as a linear function of $\theta$:

$$ r W^b = b + \beta \frac{1}{1 - \beta} \theta c. \quad (A-10) $$

Combining the firm-level equations with the reservation wage equation and dropping the firm subscript, we derive an implicit expression for the policy function:

$$ \Gamma_I (N, \theta) \equiv \int_0^1 x^{\frac{1}{2} - 1} F' (N x) dx - \beta b - \frac{\beta}{1 - \beta} \frac{c}{\lambda_m (\theta)} [r + \lambda_s + \beta p (\theta)] = 0. \quad (A-11) $$

Rearranging gives (23).

**Part (iii):** $\Gamma_I$ defines a strictly downward sloping policy function in $\theta$ - $N$-space. For $\theta \downarrow 0$, $\frac{\partial}{\partial \theta} \Gamma_I \downarrow 0$. 

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33To ensure convergence of integrals, as in Cahuc et al. (2008), $N_i^j F^{(j)}(N_i)$ is assumed to be continuous in zero ($F^{(j)}$ denotes the $j$th derivative of $F$).

34I.e., $w_i \to \infty$ for $N_i \to 0$ is imposed. This particular solution yields a wage equation that coincides with the Stole-Zwiebel solution.
\( N \downarrow N_0 > 0 \) where \( N_0 \) is determined by \( \int_0^1 x^{\frac{\beta}{\gamma} - 1} F'(N_0 x) \, dx = \beta b \). It follows that a unique intersection with the Beveridge Curve (which starts at the origin and is strictly increasing) exists for some \( \theta, N > 0 \). ■

**Proof of Lemma 3: Size of the Wage Externality.**

Given \( N_i \), the absolute difference in wage savings due to hiring an additional worker is

\[
\delta \equiv |\delta^I| - |\delta^C| = \frac{\beta}{N_i} \left[ F'(N_i) - \frac{F(N_i)}{N_i} - \frac{N_i}{\beta} \int_0^1 x^{\frac{\beta}{\gamma}} F''(N_i x) \, dx \right]. \tag{A-12}
\]

With \( \alpha' = 0 \) and \( F'' < 0 \), it holds that \( F = \kappa N_i^\alpha \) with \( \kappa > 0 \) and \( 0 < \alpha < 1 \). (A-12) becomes

\[
\delta = -\beta N_i^{\alpha - 2} (1 - \alpha)^2 \frac{1 - \beta}{1 - (1 - \alpha) \beta} \quad (< 0). \tag{A-13}
\]

Hence, \(|\delta^I| < |\delta^C|\). ■

This completes the proof. For interpretation, note that \(|F''| < |(F/N)'|\). To see this, differentiate the definition of \( \alpha(N) \), which after rearranging gives

\[
F'' = \alpha' F/N + \alpha(F/N)', \tag{A-14}
\]

and make use of the fact that \( F' < F/N: F'' = \alpha' F/N + \alpha(F/N)' > \alpha' F' + \alpha(F/N)' \). It follows that

\[
\alpha' < \frac{F'' - \alpha(F/N)'}{F'} \tag{A-15}
\]

Combining (A-14) and (A-15) we know that

\[
F'' = \alpha' F/N + \alpha(F/N)' < \frac{F'' - \alpha(F/N)'}{F'} F/N + \alpha(F/N)' \iff \frac{F''}{(F/N)'} < \alpha < 1.
\]

Accordingly, \( F'' > (F/N)' (= (F' N - F)/N^2 < 0) \) so that \(|F''| < |(F/N)'|\), which provides the intuition for why the wage dampening effect of employing a marginal individual is larger under collective bargaining that under individual bargaining.

**Proof of Lemma 4: Employment and Wages at the Firm Level.**

As \( N_i \int_0^1 x^{\frac{\beta}{\gamma}} F''(N_i x) \, dx < 0 \), \( F'(N_i^I) < F'(N_i^C) \). Concavity completes the proof.
For wages, rearranging (16) yields
\[ w_C^i = rW^b + \frac{\beta}{1 - \beta \lambda_m(\theta)} c + \beta \left[ \frac{1}{\alpha(N^C_i)} - 1 \right] \left[ rW^b + \frac{r + \lambda_s}{r + \lambda_s} c \right]. \]
(A-16)

Concavity implies \( \alpha(N^C_i) < 1 \). Comparing this expression with (25) completes the proof. ■

Proof of Proposition 2: Equilibrium Allocation under Both Regimes.
Combining the aggregate policy function under both bargaining regimes, we derive the “vertical distance” between the two policy functions at any given \( \theta \):

\[ \frac{1}{\beta} \int_0^1 x^{\frac{1}{\beta} - 1} F'(N^I x) \, dx = F'(N^C) - \frac{\beta p(\theta)}{r + \lambda_s} \left[ \frac{F(N^C)}{N^C} - F'(N^C) \right]. \]
(A-17)

Using (A-17) and \( \beta p(\theta) > 0 \) implies

\[ F'(N^C) - \frac{1}{\beta} \int_0^1 x^{\frac{1}{\beta} - 1} F'(N^I x) \, dx = \frac{\beta p(\theta)}{r + \lambda_s} \left[ \frac{F(N^C)}{N^C} - F'(N^C) \right] > 0. \]

Hence,

\[ F'(N^C) > \frac{1}{\beta} \int_0^1 x^{\frac{1}{\beta} - 1} F'(N^I x) \, dx = F'(N^I) - N^I \int_0^1 x^{\frac{1}{\beta}} F''(N^I x) \, dx \]
\[ \Leftrightarrow F'(N^C) - F'(N^I) > -N^I \int_0^1 x^{\frac{1}{\beta}} F''(N^I x) \, dx > 0. \]

Concavity implies \( N^C < N^I \). The strictly positively sloped Beveridge Curve completes the proof. The results for labor market market tightness are derived analogously. ■

Proof of Proposition 3: Efficiency.
Fix some arbitrary \( N \). At this level of employment the left-hand side of (29) is smaller than that of (30). This follows from the fact that \(-F'(N) + \frac{1}{\beta} \int_0^1 x^{\frac{1}{\beta} - 1} F'(Nx) \, dx = -\int_0^1 x^{\frac{1}{\beta}} F''(Nx) \, dx\). Finally, the left-hand side of (29) is larger than that of (31). This is a direct consequence of \( F'' < 0 \). In \( \theta-N \)-space, the policy function of the planner will be strictly below the policy function derived under individual bargaining and strictly above the policy function derived under collective bargaining (the right-hand side of the policy func-
tions is strictly increasing in $\theta$). The strictly positively sloped Beveridge Curve completes the proof. If $F'' = 0$, the policy functions are identical and as such are employment and the labor market tightness. ■